COMPLEX ANALYSIS I

Exercise 2, spring 2011

1. Show that $|z_1 - z_2| = |1 - \bar{z}_1 z_2|$, for all $z_1, z_2 \in \mathbb{C}$ with $|z_1| = 1$ or $|z_2| = 1$.

2. Let $a_1, a_2, \cdots, a_n \in \mathbb{R}$; $a_0 > a_1 > a_2 > \cdots > a_n > 0$ and $p(z) = a_0 + a_1 z + \cdots + a_n z^n, z \in \mathbb{C}$. Suppose that $p(z_0) = 0$. Show that $|z_0| > 1$.

3. Find the polarcoordinates of $z \in \mathbb{C}$ when

   a) $z = -3i$, b) $z = \sqrt{3} - i$, c) $z = 2 - i\sqrt{12}$.

4. Calculate $(1 - i\sqrt{3})^{15}$ and $(1 + i)^{11}$ and $\dfrac{(1+i)^5}{(1-i\sqrt{3})^7}$.

5. Let $z \in \mathbb{C}, |z| = 1, z \neq -1$. Show that z can be given in the form $z = \dfrac{1+it}{1-it}$ with $t \in \mathbb{R}$.

6. Find the solutions

   a) $z^4 = -1$, b) $z^6 = 1$, c) $z^3 = -i$.