COMPLEX ANALYSIS I

Exercise 3, spring 2011

1. Let $\{z \in \mathbb{C} | \; |z - z_0| > r\}$. Show that A is open.

2. Let $A = \{i, \frac{i}{2}, \frac{i}{3}, \cdots\} \subset \mathbb{C}$. Is A bounded, closed, open? Find A^0, A' and $cl(A)$.

3. Find the line running through the points $1 + i$ and $-3 + 2i$
   a) in a parametric form,
   b) in the form $ax + by = d, \; a, b, d \in \mathbb{R}$,
   c) in the form $\bar{a}z + \alpha\bar{z} = \gamma, \alpha \in \mathbb{C}$ ja $\gamma \in \mathbb{R}$.
   Find also a path joining the points $1 + i, -3 + 2i$.

4. Find the limits (if they exist)

   a) $\lim\limits_{n\to\infty} \dfrac{i^n}{n}$, b) $\lim\limits_{n\to\infty} i^n$, c) $\lim\limits_{n\to\infty} \dfrac{(1+i)^n}{n}$, d) $\lim\limits_{n\to\infty} \dfrac{2n - in^2}{(1+i)n^- 1}$.

5. Let $(a_n)\mathbb{C}$ be a sequence with $\lim\limits_{n\to\infty} a_n = a$. Show that $(a_n)_{n=1}^{\infty}$ is bounded.

6. Show that $\lim\limits_{n\to\infty} \left(1 + \frac{z}{n}\right)^n = e^x(\cos y + i \sin y)$, when $z = x + iy \in \mathbb{C}$.