COMPLEX ANALYSIS I

Exercise 4, spring 2011

1. Let (z_n) be a sequence with $z_0 = 3$ and $z_{n+1} = \frac{1}{3}z_n + 2i$. Show that (z_n) has a limit and find it.

2. Find which of the following functions are bijenctions $\mathcal{M}(f) \to \mathcal{A}(f)$ and find $f^{-1} : \mathcal{A}(f) \to \mathcal{M}(f)$ (if possible).

   a) $f(z) = \bar{z} + i,\ z \in \mathbb{C}$,

   b) $f(z) = \frac{1}{z},\ z \in \mathbb{C} \setminus \{0\}$,

   c) $f(z) = z^2 + i,\ z \in \mathbb{C}$,

   d) $f(z) = z^2 + i,\ z \in S[0, \pi)$.

3. Let $f : S[0, \frac{2\pi}{3}) \to \mathbb{C}$ a function with $f(z) = z^3 + i,\ z \in S[0, \frac{2\pi}{3})$. Show that f is a bijection $\mathcal{M}(f) \to \mathbb{C}$ and find $f^{-1}(1)$.

4. Give the function $f(z) = f(x + iy)$ in the form $f(z) = u(x, y) + iv(x, y), z \in \mathcal{M}(f)$, when

   a) $f(z) = z^3, z \in \mathbb{C}$, b) $f(z) = \frac{1}{z^2}, z \neq 0$, c) $f(z) = e^{iz}, z \in \mathbb{C}$.

5. Show that the limit $\lim\limits_{z \to z_0} f(z) = a$ of the function f is unique.

6. Find the function $f(z)$ limits in $z = 0$, when

   a) $f(z) = \frac{Rez}{z}$, b) $f(z) = \frac{z}{|z|}$, c) $f(z) = \frac{zRez}{|z|}$.