COMPLEX ANALYSIS I

Exercise 7, spring 2011

1. Show that the function
$$f(z) = \sin z$$
satisfies Cauchy-Riemann equations.

2. Let f be analytic on a region $A \subset \mathbb{C}$.
   a) Suppose that $f'(z) = 0$ for all $z \in A$.
   Show that f is constant in A.
   b) Suppose that $f = u + iv$ and u is constant in A. Show that f is constant in A. Check also the case where $u^2 + v^2$ is constant in A.

3. Find $f'(z)$, when

   a) $f(z) = \cos(z^2 + iz)$, b) $f(z) = e^{\frac{1}{z}}$.

4. Find

   a) $\log(-4)$, b) $\log 3i$, c) i^{2i}, d) i^{-i}.

5. Solve the equations
   a) $e^z = 2 + i$, b) $\sin z = i$, c) $\cos z = 0$.

6. Find the limits

   a) $\lim_{z\to 0} \dfrac{e^{z^2} - 1}{z^2 + 2z}$, b) $\lim_{z\to \frac{\pi}{2}} \dfrac{\cos z}{z - \frac{\pi}{2}}$, c) $\lim_{z\to 0} \dfrac{\cos 2z - 1}{\sin^2 z}$.

7. Show that $\sin \bar{z} = \overline{\sin z}$, $z \in \mathbb{C}$.