Complex Analysis II

Exercise 1, Spring 2011

1. Calculate
$$\int_\gamma \mathrm{Re} z dz \quad \text{and} \quad \int_\gamma \mathrm{Im} z dz,$$
where γ is
a) the line segment from $1+i$ to $2-i$.
b) The semicircle $|z|=1,\ 0 \leqslant \arg z \leqslant \pi$, with initial point 1.
c) a-centered circle with radius $r,\ \ a \in \mathbb{C},\ r>0$.

2. Evaluate $\int_\gamma |z|^2 dz$ and $\int_\gamma |z|^2 |dz|$, where γ is the square $0 \to 1 \to 1+i \to i \to 0$.

3. Let $w \in \mathbb{C}$ and $f : \mathbb{C}\setminus\{w\} \to \mathbb{C},\ f(z) = \frac{1}{z-w}$. Calculate $\int_\gamma f(z)dz$, where $\gamma = \{\ w + re^{it} \mid t \in [0, 2\pi]\ \}$, $r>0$. Does f have an antiderivative i.e is there a function F such that $F' = f$?

4. Find the antiderivatives of the following functions:

a)	$f(z) = \sin z \cos z$	b)	$f(z) = \sin 2z \cos z$
c)	$f(z) = ze^{2z}$	d)	$f(z) = z^2 \sin z$

Evaluate also $\int_\gamma e^z \sin z dz$, where $\gamma = \{2\pi \cos t + it^5 e^{t^3} \sin t \mid t \in [0,\pi]\ \}$.

5. Let γ be a piecewise smooth curve and f a continuous function on γ. Prove that
$$\int_{-\gamma} f(z)dz = -\int_\gamma f(z)dz.$$

6. Let the assumptions be as in ex. 5. Show that
$$\left|\int_\gamma f(z)dz\right| \leqslant \int_\gamma |f(z)|\,|dz|.$$
HINT: Start by assuming that γ is smooth and choosing an appropriate constant $c \in \mathbb{C}$ such that $\left|\int_\gamma f(z)dz\right| = \int_\gamma cf(z)dz$.