Complex Analysis II

Exercise 2, Spring 2011

1. Let γ be the unit circle. Calculate $\int_\gamma f(z)dz$, when

   a) $f(z) = \frac{1}{z^2+2z+2}$, b) $f(z) = \tan z$

2. Evaluate

$$\int_\gamma \frac{1}{(z-z_0)^n}dz,\ n = 2, 3...$$

   where γ is a piecewise smooth Jordan-curve with
   a) $z_0 \in I(\gamma)$ (the interior of γ)
   b) $z_0 \in E(\gamma)$ (the exterior of γ.)

3. Show that

$$\int_0^{2\pi} e^{\cos t} \cos(t + \sin t)dt = 0 \quad \text{and} \quad \int_0^{2\pi} e^{\cos t} \sin(t + \sin t)dt = 0.$$

4. Calculate

$$\int_\gamma \frac{1}{1+z^2}dz,$$

   where γ is the circle with
   a) center i, radius 1, b) center $-i$, radius 1, c) center 0, radius 2.
   Hint: partial fraction decomposition.

5. Show that the sequence of triangles $\Delta, \Delta_1, \Delta_2, ..., \Delta_n, ...$ constructed in the proof of the Cauchy-Goursat Theorem have the following properties:

   a) $\frac{|\partial\Delta_n|^2}{|\Delta_n|} = \frac{|\partial\Delta|^2}{|\Delta|}$ for every $n \in \mathbb{Z}_+$.

   b) If for every $n \in \mathbb{Z}_+$ we choose an arbitrary point $z_n \in \Delta_n$, then the sequence (z_n) converges to some point z_0.

   c) Given $\delta > 0$, there exists $N \in \mathbb{Z}_+$ such that

$$\Delta_n \subset D_\delta(z_0) \quad \text{for every } n \geqslant N.$$

   d) The intersection $\cap_{n=1}^{\infty}\Delta_n$ consists of exactly one point.