

COMPLEX ANALYSIS II

Exercise 3, Spring 2011

1. Calculate the following integrals:

- a) $\int_{\gamma} \frac{\sin z}{z-i} dz$, where $\gamma = \{2e^{it} \mid t \in [0, 2\pi]\}$,
b) $\int_{\gamma} \frac{\cosh z}{z-\pi i} dz$, where $\gamma = \{4e^{2\pi it} \mid t \in [0, 1]\}$,
c) $\int_{\gamma} \frac{\cos z}{z-2\pi} dz$, where $\gamma = \{3 + \pi e^{it^2} \mid t \in [0, \sqrt{2\pi}]\}$.

2. Calculate

$$\int_{\gamma} \frac{e^z}{z(z-2i)} dz,$$

where

a) $\gamma = \{e^{it} \mid t \in [0, 2\pi]\}$, b) $\gamma = \{3e^{it} \mid t \in [0, 2\pi]\}$.

3. Evaluate

$$\int_0^{2\pi} e^{a \cos t} \cos(a \sin t) dt,$$

where $a \in \mathbb{R} \setminus \{0\}$.

4. Calculate

a) $\int_{\gamma} \frac{e^{az}}{z^2+1} dz$, b) $\int_{\gamma} \frac{e^{az}}{(z^2+1)^2} dz$,

where $\gamma = \{3e^{it} \mid t \in [0, 2\pi]\}$ and $a \in \mathbb{R} \setminus \{0\}$.

5. Calculate

a) $\int_{\gamma} \frac{e^{iz}}{z^3} dz$, b) $\int_{\gamma} \frac{\sin z}{z^{n+1}} dz$,

where $\gamma = \{2e^{it} \mid t \in [0, 2\pi]\}$ and $n \in \mathbb{N}$.

6. Let f be an analytic function in a domain A and D an open disk such that $cl(D) \subset A$. Prove that for all $z \in D$,

$$\lim_{h \rightarrow 0} \int_{\partial D} \frac{f(w)}{(w-(z+h))^m (w-z)^n} dw = \int_{\partial D} \frac{f(w)}{(w-z)^{m+n}} dw$$

for all $m, n \in \mathbb{Z}_+$.

Hint: Since $|f|$ is continuous on ∂D , which is closed (as a set) and bounded (i.e compact), there exists $M > 0$ such that $|f(w)| \leq M$ for all $w \in \partial D$.