Complex analysis II

Exercise 7, Spring 2011

1. Find the Laurent-series for the function

$$f(z) := \frac{1}{z(z+1)(z+2)}$$

on domain a) $0 < |z| < 1$, b) $1 < |z| < 2$, c) $|z| > 2$.

2. Find the Laurent-series of the following functions on domain $|z| > 0$ and determine the type of singularity at 0, when

$$\text{a)}\ \ f(z) = \frac{1-\cos z}{z}, \quad \text{b)}\ \ f(z) = \frac{e^{z^2}}{z^3}.$$

3. Determine the singularities of f and the residues on those points, when

$$\text{a)}\ \ f(z) = \frac{2z+1}{z^2-z-2}, \quad \text{b)}\ \ f(z) = \frac{z^2+4}{z^3+2z^2+2z}, \quad \text{c)}\ \ f(z) = \frac{1}{\cos z}.$$

4. Calculate

$$\text{a)}\ \ \int_{\partial D_2(0)} \frac{e^z}{z^3+z}dz \quad \text{b)}\ \ \int_{\partial D_r(0)} \frac{\sin z}{(z-a)(z-b)}dz,$$

where $r > 0$ and $|a|, |b| < r$.

5. Let γ be a piecewise smooth positively oriented Jordan curve. Suppose f is an analytic function on a domain that contains the set $I(\gamma) \cup \gamma$. Show that

$$f(z) = \frac{1}{2\pi i}\int_\gamma \frac{f(w)}{w-z}dw \quad \text{for all } z \in I(\gamma).$$