MPTT 2, 2. HARJOITUS

2. c) $\begin{vmatrix} -2 & -1 & 4 \\ 6 & -3 & -2 \\ 4 & 1 & 2 \end{vmatrix} = \begin{vmatrix} -10 & -3 & 0 \\ 10 & -2 & 0 \\ 4 & 1 & 2 \end{vmatrix} = 0 + 0 + 2 \cdot (-1)^{3+3} \begin{vmatrix} -10 & -3 \\ 10 & -2 \end{vmatrix}$

$= 2 \cdot (-10 \cdot (-2) - (-3) \cdot 10) = 2 \cdot 50 = 100$

d) $\begin{vmatrix} 3 & -2 & -5 & 4 \\ -5 & 2 & 8 & -5 \\ -2 & 4 & 7 & -3 \\ 2 & -3 & -5 & 8 \end{vmatrix} = \begin{vmatrix} 3 & -2 & -5 & 4 \\ -2 & 0 & 3 & -1 \\ 4 & 0 & -3 & 5 \\ 2 & -3 & -5 & 8 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 & -4 \\ -2 & 0 & 3 & -1 \\ 4 & 0 & -3 & 5 \\ 2 & -3 & -5 & 8 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 0 & -4 \\ -2 & 0 & 3 & -1 \\ 4 & 0 & -3 & 5 \\ 5 & 0 & -5 & -4 \end{vmatrix} = 1 \cdot (-1)^{1+2} \begin{vmatrix} -2 & 3 & -1 \\ 4 & -3 & 5 \\ 5 & -5 & -4 \end{vmatrix} = -1 \cdot \begin{vmatrix} -2 & 3 & -1 \\ -6 & 12 & 0 \\ 13 & -17 & 0 \end{vmatrix}$

$= -1 \cdot (-1) \cdot (-1)^{1+3} \begin{vmatrix} -6 & 12 \\ 13 & -17 \end{vmatrix} = -6 \cdot (-17) - 12 \cdot 13 = 102 - 156 = -54$

e) $\begin{vmatrix} 1 & 1 & 2 & 3 \\ 2 & 2 & -1 & 0 \\ 3 & 2 & 1 & 1 \\ 3 & 2 & 6 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & 0 & -5 & -6 \\ 0 & -1 & -5 & -8 \\ 0 & -1 & 0 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & 0 & -5 & -6 \\ 0 & -1 & 0 & 2 \\ 0 & -1 & 0 & 2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & 0 & -5 & -6 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{vmatrix} = 0$

Kaksi samaa riviä
$\Rightarrow$ saadaan nollarivi $\Rightarrow$ kun kehitetään tämän nollarivin mukaan tulee determinantiksi nolla.

f) $\begin{vmatrix} 3 & 1 & 1 & 7 \\ 5 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \nexists$

Ei voida laskea, koska kyseessä ei ole neliömatriisi.

③ $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ $\det A = 2 \cdot 3 - 1 \cdot 5 = 1 \neq 0 \Rightarrow A^{-1} \exists$

b) $K = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$, $A_{ij} = (-1)^{i+j} |M_{ij}|$

$A_{11} = (-1)^{1+1} \cdot 3 = 3$ $A_{21} = (-1)^{1+2} \cdot 5 = -5$

$A_{12} = (-1)^{2+1} \cdot 1 = -1$ $A_{22} = (-1)^{2+2} \cdot 2 = 2$

$\Rightarrow K = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

Nyt $A^{-1} = \dfrac{1}{\det A} K^T = \dfrac{1}{1} \cdot \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$

c) $(A \mid I) = \left(\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right) \overset{-1}{} \sim \left(\begin{array}{cc|cc} 1 & 2 & 1 & -1 \\ 1 & 3 & 0 & 1 \end{array}\right) \overset{-1}{}$

$\sim \left(\begin{array}{cc|cc} 1 & 2 & 1 & -1 \\ 0 & 1 & -1 & 2 \end{array}\right) \overset{-2}{} \sim \left(\begin{array}{cc|cc} 1 & 0 & 3 & -5 \\ 0 & 1 & -1 & 2 \end{array}\right) = (I \mid A^{-1})$

Eli $A^{-1} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$

④ $A = \begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix}$

$\det A = \begin{vmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{vmatrix} \overset{+2}{} = \begin{vmatrix} 1 & 0 & 0 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{vmatrix} = (-1) \cdot (-1)^{3+2} \begin{vmatrix} 1 & 0 \\ -4 & 1 \end{vmatrix} = 1 \neq 0 \Rightarrow A^{-1} \exists$

a) $A_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 1 \\ -1 & -1 \end{vmatrix} = 1$ $A_{12} = (-1)^{1+2} \begin{vmatrix} -4 & 1 \\ 6 & -1 \end{vmatrix} = 2$

$A_{13} = (-1)^{1+3} \begin{vmatrix} -4 & 0 \\ 6 & -1 \end{vmatrix} = 4$ $A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 2 \\ -1 & -1 \end{vmatrix} = 0$

$A_{22} = (-1)^{2+2} \begin{vmatrix} -11 & 2 \\ 6 & -1 \end{vmatrix} = -1$ $\qquad$ $A_{23} = (-1)^{2+3} \begin{vmatrix} -11 & 2 \\ 6 & -1 \end{vmatrix} = 1$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 2 \\ 0 & 1 \end{vmatrix} = 2$ $\qquad$ $A_{32} = (-1)^{3+2} \begin{vmatrix} -11 & 2 \\ -4 & 1 \end{vmatrix} = 3$

$A_{33} = (-1)^{3+3} \begin{vmatrix} -11 & 2 \\ -4 & 0 \end{vmatrix} = 8$

$$\Rightarrow K = \begin{pmatrix} 1 & 2 & 4 \\ 0 & -1 & 1 \\ 2 & 3 & 8 \end{pmatrix} \quad \Rightarrow A^{-1} = \frac{1}{\det A} K^T = 1 \cdot \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$$

b) $$(A \,\vdots\, I) = \left(\begin{array}{ccc|ccc} -11 & 2 & 2 & 1 & 0 & 0 \\ -4 & 0 & 1 & 0 & 1 & 0 \\ 6 & -1 & -1 & 0 & 0 & 1 \end{array}\right) +2 \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ -4 & 0 & 1 & 0 & 1 & 0 \\ 6 & -1 & -1 & 0 & 0 & 1 \end{array}\right) +4 \; -6$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 & 1 & 8 \\ 0 & -1 & -1 & -6 & 0 & -11 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & -1 & -1 & -6 & 0 & -11 \\ 0 & 0 & 1 & 4 & 1 & 8 \end{array}\right) +1$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & -1 & 0 & -2 & 1 & -3 \\ 0 & 0 & 1 & 4 & 1 & 8 \end{array}\right) \cdot (-1) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 2 & -1 & 3 \\ 0 & 0 & 1 & 4 & 1 & 8 \end{array}\right) = (I \,|\, A^{-1})$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$$

Tarkistus:

$$A \cdot A^{-1} = \begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I \quad \text{OK.}$$