

$$\textcircled{5} \quad \begin{cases} -11x + 2y + 2z = 1 \\ -4x + z = 2 \\ 6x - y - z = 3 \end{cases}$$

Muuttujien lkm on sama kuin yhtälöiden lkm, joten kyseessä on kolmen muuttujan x, y, z vakiokerroiminen lineaarinen kolmen yhtälön ryhmä

$$a) \quad \begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \Leftrightarrow A\bar{x} = \bar{c}$$

Tentävä 4 $\Rightarrow \det A = 1 \neq 0$, joten A on säännöllinen ja A^{-1} on olemassa

Näin ollen yhtälöryhmällä on yksikäsitteinen ratkaisu $\bar{x} = A^{-1}\bar{c}$

$$\text{Tent 4} \Rightarrow A^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$$

$$\bar{x} = A^{-1}\bar{c} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\Leftrightarrow \bar{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \cdot 1 + 0 \cdot 2 + 2 \cdot 3 \\ 2 \cdot 1 + (-1) \cdot 2 + 3 \cdot 3 \\ 4 \cdot 1 + 1 \cdot 2 + 8 \cdot 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \\ 30 \end{pmatrix} \Leftrightarrow \begin{cases} x = 7 \\ y = 9 \\ z = 30 \end{cases}$$

b) Voidaan käyttää Cramerin sääntöä, koska $\det A = 1 \neq 0$ ja yhtälöryhmässä on 3 tunnetonta ja 3 yhtälöä, jolloin yhtälöryhmällä on yksikäsitteinen ratkaisu $\vec{x} = A^{-1} \vec{c}$

$$A = \begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix}$$

$$x = \frac{1}{|A|} \begin{vmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 3 & -1 & -1 \end{vmatrix} \stackrel{-1}{=} \frac{1}{|A|} \begin{vmatrix} 1 & 2 & 0 \\ 1 & 2 & 0 \\ 3 & -1 & 0 \end{vmatrix} \stackrel{\downarrow}{=} 1 \cdot 1 \cdot (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = -(-1-6) = 7$$

$$y = \frac{1}{|A|} \begin{vmatrix} -11 & 1 & 2 \\ -4 & 2 & 1 \\ 6 & 3 & -1 \end{vmatrix} \stackrel{\uparrow_1}{=} \frac{1}{|A|} \begin{vmatrix} 1 & 7 & 0 \\ 2 & 5 & 0 \\ 6 & 3 & -1 \end{vmatrix} = 1 \cdot (-1) \cdot (-1)^{3+3} \begin{vmatrix} 1 & 7 \\ 2 & 5 \end{vmatrix} = - (5 - 14) = 9$$

$$z = \frac{1}{|A|} \begin{vmatrix} -11 & 2 & 1 \\ -4 & 0 & 2 \\ 6 & -1 & 3 \end{vmatrix} \stackrel{\uparrow_2}{=} \frac{1}{|A|} \begin{vmatrix} 1 & 0 & 7 \\ 1 & -4 & 0 \\ 6 & -1 & 3 \end{vmatrix} = 1 \cdot (-1) \cdot (-1)^{3+2} \begin{vmatrix} 1 & 7 \\ -4 & 2 \end{vmatrix} = 2 + 28 = 30$$

$$\Rightarrow \begin{cases} x = 7 \\ y = 9 \\ z = 30 \end{cases}$$

c) Täydennetty kerroinmatriisi

$$(A|c) = \left(\begin{array}{ccc|c} -11 & 2 & 2 & 1 \\ -4 & 0 & 1 & 2 \\ 6 & -1 & -1 & 3 \end{array} \right) \stackrel{2}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ -4 & 0 & 1 & 2 \\ 6 & -1 & -1 & 3 \end{array} \right) \stackrel{4}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ -4 & 0 & 1 & 2 \\ 6 & -1 & -1 & 3 \end{array} \right) \stackrel{-6}{\sim}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 30 \\ 0 & -1 & -1 & -39 \end{array} \right) \stackrel{2}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & -1 & -1 & -39 \\ 0 & 0 & 1 & 30 \end{array} \right) \stackrel{1}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & -1 & 0 & -9 \\ 0 & 0 & 1 & 30 \end{array} \right) \cdot (-1)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 30 \end{array} \right) \Rightarrow \begin{cases} x = 7 \\ y = 9 \\ z = 30 \end{cases}$$

$$d) \begin{cases} -11x + 2y + 2z = 1 \\ -4x \quad \quad + z = 2 \\ 6x - y - z = 3 \end{cases}$$

$$\begin{cases} -11x + 2y + 2z = 1 \\ 6x - y - z = 3 \quad | \cdot 2 \end{cases}$$

$$\begin{cases} -11x + 2y + 2z = 1 \\ + 12x - 2y - 2z = 6 \end{cases} \Rightarrow x = 7$$

$$\Rightarrow \begin{cases} -4 \cdot 7 + z = 2 \\ z = 30 \end{cases}$$

$$6 \cdot 7 - y - 30 = 3$$

$$-y = -9$$

$$y = 9$$

$$\Rightarrow \begin{cases} x = 7 \\ y = 9 \\ z = 30 \end{cases}$$

$$6) a) \begin{cases} 3x + 4y - 3z = -3 \\ 2x + 3y + 2z = 5 \\ x + y + z = 4 \\ 3x + 4y + 3z = 9 \end{cases}$$

$$(A|C) = \left(\begin{array}{ccc|c} 3 & 4 & -3 & -3 \\ 2 & 3 & 2 & 5 \\ 1 & 1 & 1 & 4 \\ 3 & 4 & 3 & 9 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 2 & 3 & 2 & 5 \\ 3 & 4 & -3 & -3 \\ 3 & 4 & 3 & 9 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 1 & -6 & -15 \\ 0 & 1 & 0 & -3 \end{array} \right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 7 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & -6 & -12 \\ 0 & 0 & 0 & 0 \end{array} \right) \cdot \left(-\frac{1}{6} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 7 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\Rightarrow \begin{cases} x = 5 \\ y = -3 \\ z = 2 \\ 0 = 0 \end{cases}$$

$$c) \begin{cases} x + 2y - z + 3w = 3 \\ 2x + 4y + 4z + 3w = 9 \\ 3x + 6y - z + 8w = 10 \end{cases}$$

$$(A|c) = \left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 3 \\ 2 & 4 & 4 & 3 & 9 \\ 3 & 6 & -1 & 8 & 10 \end{array} \right) \xrightarrow{-2} \sim \left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 3 \\ 0 & 0 & 6 & -3 & 3 \\ 0 & 0 & 2 & -1 & 1 \end{array} \right) \xrightarrow{-3} \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 5 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 1 \end{array} \right) \xrightarrow{\uparrow} \sim \left(\begin{array}{cccc|c} 1 & 2 & 5 & 0 & 6 \\ 0 & 0 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\Rightarrow \begin{cases} x + 2y + 5z = 6 \\ 2z - w = 1 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} z = \frac{1}{5}(6 - x - 2y) \\ w = 2z - 1 \end{cases}$$

$$\Rightarrow \begin{cases} z = \frac{1}{5}(6 - x - 2y) \\ w = 2z - 1 \\ x \in \mathbb{R} \\ y \in \mathbb{R} \end{cases}$$