MPTT 2, vko 11

(22) a) $\int \frac{x+1}{\sqrt[3]{x^2+2x+2}}\,dx$ | $D(x^2+2x+2)$ $= 2x+2$ $= 2(x+1)$

$= \int (x+1)(x^2+2x+2)^{-\frac{1}{3}}\,dx$

$= \frac{1}{2}\int 2(x+1)(x^2+2x+2)^{-\frac{1}{3}}\,dx$

$= \frac{1}{2}\cdot\frac{(x^2+2x+2)^{\frac{2}{3}}}{2/3} + C = \frac{3}{4}(x^2+2x+2)^{\frac{2}{3}} + C$

b) $\int x e^{x^2}\,dx$ | $D x^2 = 2x$

$= \frac{1}{2}\int 2x e^{x^2}\,dx = \frac{1}{2}e^{x^2} + C$

c) $\int \frac{x}{x^2 \ln x^2}\,dx$ | $D \ln x^2 = \frac{1}{x^2}\cdot 2x = \frac{2}{x}$

$= \int \frac{1}{x\ln x^2}\,dx = \int \frac{1}{x}\cdot\frac{1}{\ln x^2}\,dx = \int \frac{\frac{1}{x}}{\ln x^2}\,dx$

$= \frac{1}{2}\int \frac{\frac{2}{x}}{\ln x^2}\,dx = \frac{1}{2}\ln|\ln x^2| + C$

d) $\int 2^{x^2+1} x\,dx$ | $D(x^2+1) = 2x$

$= \frac{1}{2}\int 2x\, 2^{x^2+1}\,dx = \frac{1}{2}\,\frac{2^{x^2+1}}{\ln 2} + C$

$= 2^{-1}\cdot\frac{2^{x^2+1}}{\ln 2} + C = \frac{2^{x^2}}{\ln 2} + C$

e) $\int \frac{x+1}{2x^2+4x+5}\,dx$ | $D(2x^2+4x+5) = 4x+4 = 4(x+1)$

$= \frac{1}{4}\int \frac{4x+4}{2x^2+4x+5}\,dx = \frac{1}{4}\ln|2x^2+4x+5| + C$

(23) a) $\int x^2 |x| \, dx$ $\quad |x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$

$$= \begin{cases} \int x^2 \cdot x \, dx & , x \geq 0 \\ \int x^2 \cdot (-x) \, dx & , x < 0 \end{cases}$$

$$= \begin{cases} \int x^3 \, dx & , x \geq 0 \\ \int -x^3 \, dx & , x < 0 \end{cases}$$

$$= \begin{cases} \frac{x^4}{4} + C & , x \geq 0 \\ -\frac{x^4}{4} + C & , x < 0 \end{cases} = \frac{1}{4} x^3 |x| + C$$

b) $$\int \frac{dx}{1+e^x} = \int \frac{1}{1+e^x} \, dx = \int \frac{1+e^x-e^x}{1+e^x} \, dx$$

$$= \int \frac{1+e^x}{1+e^x} \, dx - \int \frac{e^x}{1+e^x} \, dx$$

$$= \int 1 \, dx - \int \frac{e^x}{1+e^x} \, dx$$

$$= x - \ln \underbrace{|1+e^x|}_{>0} + C = x - \ln(1+e^x) + C$$

c) $$\int \frac{\ln x}{x} \, dx = \int \ln x \cdot \underbrace{\frac{1}{x}}_{= D \ln x} \, dx$$

$$= \int (\ln x)' \cdot \frac{1}{x} \, dx = \frac{(\ln x)^2}{2} + C$$

24 a) $\int (x^2 - \sqrt{x} + 2)\,dx = \int (x^2 - x^{\frac{1}{2}} + 2)\,dx$

$= \frac{x^3}{3} - \frac{x^{\frac{3}{2}}}{3/2} + 2x + C = \frac{x^3}{3} - \frac{2}{3}x\sqrt{x} + 2x + C$

b) $\int \sqrt{2+5x}\,dx = \int (2+5x)^{\frac{1}{2}}\,dx$

$= \frac{1}{5}\int 5(2+5x)^{\frac{1}{2}}\,dx = \frac{1}{5} \cdot \frac{(2+5x)^{\frac{3}{2}}}{3/2} + C$

$= \frac{2}{15}(2+5x)^{\frac{3}{2}} + C$

c) $\int \frac{dx}{(3x+2)^2} = \int \frac{1}{(3x+2)^2}\,dx$

$= \int (3x+2)^{-2}\,dx = \frac{1}{3}\int 3(3x+2)^{-2}\,dx$

$= \frac{1}{3} \cdot \frac{(3x+2)^{-1}}{-1} + C = -\frac{1}{3(3x+2)} + C$

$= -\frac{1}{9x+6} + C$

d) $\int \frac{x^3 - 1}{x - 1}\,dx$

$= \int \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}\,dx$

$= \int (x^2 + x + 1)\,dx$

$= \frac{x^3}{3} + \frac{x^2}{2} + x + C$

$$
\begin{array}{r|l}
 & x^2 + x + 1 \\
\hline
x - 1 & x^3 \qquad -1 \\
 & -x^3 \pm x^2 \\
 & x^2 \\
 & -x^2 \pm x \\
 & x \\
 & -x \pm 1 \\
 & 0
\end{array}
$$