MPTT2, viikko 15

(30) $y_1 = \sqrt{1-x}$, $y_2 = \sqrt{x-2}$, $y_3 = 1$, $y_4 = 2$

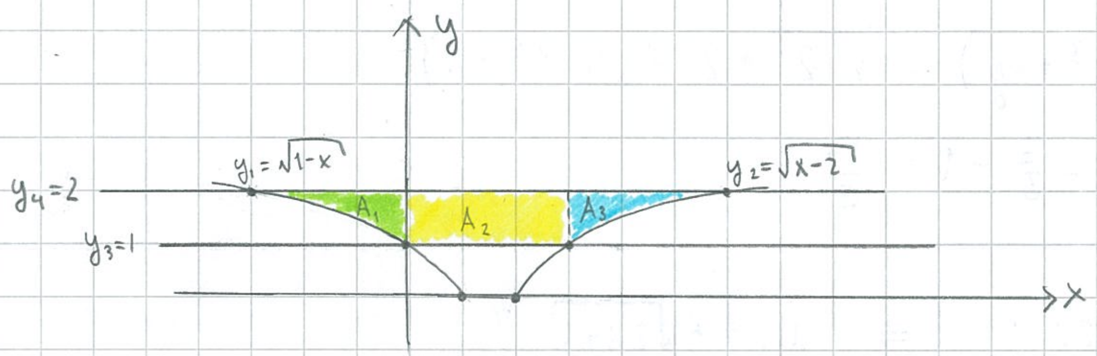
Leikkauspisteet:

$\begin{cases} y = \sqrt{1-x} \\ y = 2 \end{cases}$ $\Rightarrow x = 1 - y^2 = -3$ $(-3, 2)$

$\begin{cases} y = \sqrt{1-x} \\ y = 1 \end{cases}$ $\Rightarrow x = 0$ $(0, 1)$

$\begin{cases} y = \sqrt{x-2} \\ y = 1 \end{cases}$ $\Rightarrow x = 3$ $(3, 1)$

$\begin{cases} y = \sqrt{x-2} \\ y = 2 \end{cases}$ $\Rightarrow x = 6$ $(6, 2)$



a) x-akselin suhteen

$$A_1 = \int_{-3}^{0} (y_4 - y_1)\, dx = \int_{-3}^{0} (2 - \sqrt{1-x})\, dx = \int_{-3}^{0} (2 + (-1)(1-x)^{\frac{1}{2}})\, dx$$

$$= \Big/_{-3}^{0} \left(2x + \frac{(1-x)^{3/2}}{3/2}\right) = \Big/_{-3}^{0} \left(2x + \frac{2}{3}(1-x)^{3/2}\right)$$

$$= 2 \cdot 0 + \frac{2}{3} \cdot (1-0)^{3/2} - \left(2 \cdot (-3) + \frac{2}{3}(1-(-3))^{3/2}\right)$$

$$= \frac{2}{3} + 6 - \frac{16}{3} = \frac{2}{3} + \frac{18}{3} - \frac{16}{3} = \frac{4}{3}$$

$$A_2 = \int_{0}^{3} (y_4 - y_3)\, dx = \int_{0}^{3} (2-1)\, dx = \int_{0}^{3} 1\, dx = \Big/_{0}^{3} x = 3 - 0 = 3$$

$$A_3 = \int_{3}^{6} (y_4 - y_2)\, dx = \int_{3}^{6} (2 - \sqrt{x-2})\, dx = \int_{3}^{6} (2 - (x-2)^{1/2})\, dx$$

$$= \Big/_{3}^{6} \left(2x - \frac{(x-2)^{3/2}}{3/2}\right) = \Big/_{3}^{6} \left(2x - \frac{2}{3}(x-2)^{3/2}\right)$$

$$= 2 \cdot 6 - \frac{2}{3}(6-2)^{3/2} - \left(2 \cdot 3 - \frac{2}{3}(3-2)^{3/2}\right) = 12 - \frac{16}{3} - 6 + \frac{2}{3} = \frac{4}{3}$$

$$A = A_1 + A_2 + A_3 = \frac{4}{3} + 3 + \frac{4}{3} = \frac{17}{3}$$

b) y-akselin suhteen

$y = \sqrt{1-x} \quad \| (\,)^2$

$y^2 = 1 - x$

$\Rightarrow x_1 = 1 - y^2$

$y = \sqrt{x-2} \quad \| (\,)^2$

$y^2 = x - 2$

$\Rightarrow x_2 = y^2 + 2$

$$A = \int_1^2 (x_2 - x_1)\,dy = \int_1^2 (y^2 + 2 - (1 - y^2))\,dy = \int_1^2 (2y^2 + 1)\,dy$$

$$= \Big/_1^2 \left(\frac{2}{3}y^3 + y\right) = \frac{2}{3}\cdot 2^3 + 2 - \left(\frac{2}{3}\cdot 1^3 + 1\right)$$

$$= \frac{16}{3} + 2 - \frac{2}{3} - 1 = \frac{17}{3}$$

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31. $y^2 = 2x + 1 \quad \Rightarrow \quad y_1 = +\sqrt{2x+1}$, $y_2 = -\sqrt{2x+1}$

$x - y - 1 = 0 \quad \Rightarrow \quad y_3 = x - 1$

Leikkauspisteet.

$\begin{cases} y = \sqrt{2x+1} \\ y = x - 1 \end{cases}$

$\Rightarrow x = 4$

$\Rightarrow y = 3$

$(4, 3)$

$\begin{cases} y = -\sqrt{2x+1} \\ y = x - 1 \end{cases}$

$\Rightarrow x = 0$

$\Rightarrow y = -1$

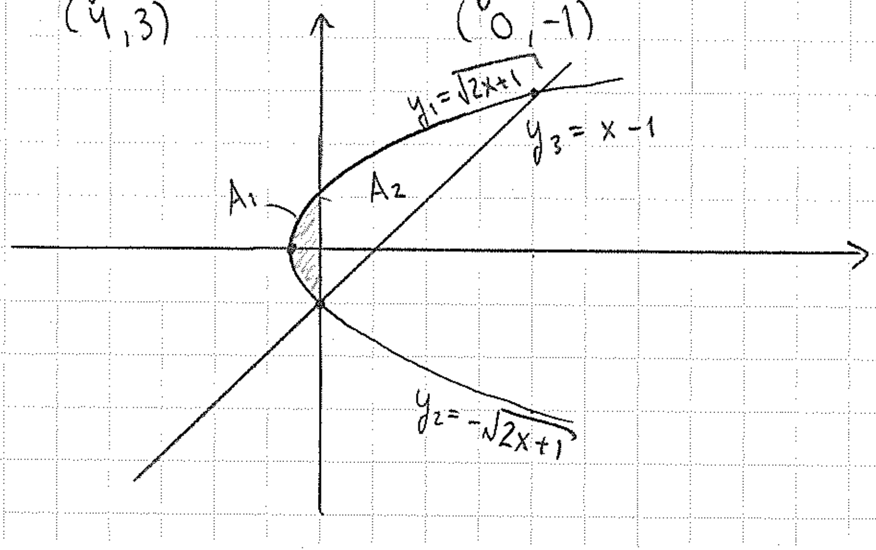
$(0, -1)$

$\begin{cases} y = \sqrt{2x+1} \\ y = -\sqrt{2x+1} \end{cases}$

$\Rightarrow y = 0$

$x = -\frac{1}{2}$

$(-\frac{1}{2}, 0)$



a) x-akselin suhteen

$$A_1 = \int_{-1/2}^{0} (y_1 - y_2)\,dx = \int_{-1/2}^{0} \left(\sqrt{2x+1} - (-\sqrt{2x+1})\right) dx$$

$$= \int_{-1/2}^{0} \left(2\sqrt{2x+1}\right) dx = \int_{-1/2}^{0} 2(2x+1)^{1/2}\,dx = \Big/_{-1/2}^{0} \frac{(2x+1)^{3/2}}{3/2}$$

$$= \Big/_{-1/2}^{0} \frac{2}{3}(2x+1)^{3/2} = \frac{2}{3}\cdot(2\cdot 0+1)^{3/2} - \frac{2}{3}\left(2\cdot\left(-\frac{1}{2}\right)+1\right)^{3/2}$$

$$= \frac{2}{3} - 0 = \frac{2}{3}$$

$$A_2 = \int_{0}^{4} (y_1 - y_3)\,dx = \int_{0}^{4} \left(\sqrt{2x+1} - (x-1)\right)dx$$

$$= \frac{1}{2}\int_{0}^{4} 2(2x+1)^{1/2}\,dx - \int_{0}^{4}(x-1)\,dx = \frac{1}{2}\Big/_{0}^{4} \frac{(2x+1)^{3/2}}{3/2} - \Big/_{0}^{4}\left(\frac{x^2}{2} - x\right)$$

$$= \frac{1}{3}\left((2\cdot 4+1)^{3/2} - (2\cdot 0+1)^{3/2}\right) - \left(\frac{1}{2}\cdot 4^2 - 4 - 0\right)$$

$$= \frac{1}{3}(27-1) - (8-4) = \frac{26}{3} - 4 = \frac{14}{3}$$

$$A = A_1 + A_2 = \frac{2}{3} + \frac{14}{3} = \frac{16}{3}$$

b) y-akselin suhteen

$y^2 = 2x+1$
$2x = y^2 - 1$
$\Rightarrow x_1 = \frac{1}{2}y^2 - \frac{1}{2}$

$x - y - 1 = 0$
$\Rightarrow x_2 = y + 1$

$$A = \int_{-1}^{3} (x_2 - x_1)\,dy = \int_{-1}^{3} \left(y + 1 - \frac{1}{2}y^2 + \frac{1}{2}\right) dy$$

$$= \int_{-1}^{3} \left(-\frac{1}{2}y^2 + y + \frac{3}{2}\right) dy = \Big/_{-1}^{3}\left(-\frac{y^3}{2\cdot 3} + \frac{y^2}{2} + \frac{3}{2}y\right)$$

$$= -\frac{3^3}{6} + \frac{3^2}{2} + \frac{3}{2}\cdot 3 - \left(-\frac{(-1)^3}{6} + \frac{(-1)^2}{2} + \frac{3}{2}(-1)\right) = \cancel{-\frac{9}{2}} + \cancel{\frac{9}{2}} + \frac{9}{2} - \frac{1}{6} - \frac{1}{2} + \frac{3}{2}$$

$$= \frac{16}{3}$$

(32)

a) ositlaisintegroinnilla

$$\int_0^1 x(1-x)^3\,dx$$

$g(x) = x$ $\qquad f'(x) = (1-x)^3$

$g'(x) = 1$ $\qquad f(x) = -\int -(1-x)^3 dx = -\frac{(1-x)^4}{4} = -\frac{1}{4}(1-x)^4$

$$= \Big/_0^1 -\frac{1}{4}(1-x)^4 \cdot x - \int_0^1 -\frac{1}{4}(1-x)^4 \cdot 1\,dx$$

$$= \Big/_0^1 -\frac{1}{4}(1-x)^4 \cdot x - \frac{1}{4} \Big/_0^1 \frac{1}{5}(1-x)^5$$

$$= -\frac{1}{4}\left((1-1)^4 \cdot 1 - (1-0)^4 \cdot 0\right) - \frac{1}{20}\left((1-1)^5 - (1-0)^5\right)$$

$$= \frac{1}{20}$$

b) sijoituksella

$$\int_0^1 x(1-x)^3\,dx$$

Sij $t = 1-x$

$x = 1-t = g(t)$

$dx = -1\,dt = g'(t)\,dt$

Rajat: $x=0 \Rightarrow t=1$

$x=1 \Rightarrow t=0$

$$= \int_1^0 (1-t) \cdot t^3 \cdot (-1)\,dt$$

$$= \int_1^0 -t^3(1-t)\,dx$$

$$= \int_1^0 (t^4 - t^3)\,dt$$

$$= \Big/_1^0 \left(\frac{t^5}{5} - \frac{t^4}{4}\right) = \frac{1}{5} \cdot 0^5 - \frac{1}{4} \cdot 0^4 - \left(\frac{1}{5} \cdot 1^5 - \frac{1}{4} \cdot 1^4\right)$$

$$= -\frac{1}{5} + \frac{1}{4} = -\frac{4}{20} + \frac{5}{20} = \frac{1}{20}$$