MPTT 2, Viikko 16

(33) Laske Taylorin sarjakehitelmän avulla $\int_0^2 e^{x^2}dx$ tarkkuudella $k=3$

$f(x) = e^{x^2}$, valitaan $a=1 \in [0,2]$

$f(x) = e^{x^2}$ $\qquad f(1) = e^{1^2} = e$

$f'(x) = 2xe^{x^2}$ $\qquad f'(1) = 2\cdot 1 e^{1^2} = 2e$

$f''(x) = 2\cdot e^{x^2} + 2x \cdot 2xe^{x^2}$

$= 2e^{x^2} + 4x^2e^{x^2}$ $\qquad f''(1) = 2e + 4\cdot 1^2 \cdot e^{1^2} = 2e + 4e = 6e$

$f'''(x) = 2\cdot 2xe^{x^2} + 8xe^{x^2} + 4x^2 \cdot 2xe^{x^2}$

$= 4xe^{x^2} + 8xe^{x^2} + 8x^3e^{x^2}$

$= 12xe^{x^2} + 8x^3e^{x^2}$ $\qquad f'''(1) = 12e + 8e = 20e$

$$\int_0^2 e^{x^2}dx \approx \int_0^2 \left(f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 \right) dx$$

$$= \int_0^2 \left(e + 2e(x-1) + \frac{6e}{2}(x-1)^2 + \frac{20e}{6}(x-1)^3 \right) dx$$

$$= \int_0^2 \left(e + 2ex - 2e + 3e(x-1)^2 + \frac{10e}{3}(x-1)^3 \right) dx$$

$$= \int_0^2 \left(-e + 2ex + 3e(x-1)^2 + \frac{10e}{3}(x-1)^3 \right) dx$$

$$= \Big/_0^2 \left(-ex + \frac{\cancel{2}ex^2}{\cancel{2}} + \frac{\cancel{3}e(x-1)^3}{\cancel{3}} + \frac{10e}{3}\frac{(x-1)^4}{4} \right)$$

$$= -e\cdot 2 + e\cdot 2^2 + e(2-1)^3 + \frac{5e}{6}(2-1)^4$$
$$- \left(-e\cdot 0 + e\cdot 0^2 + e(0-1)^3 + \frac{5e}{6}(0-1)^3 \right)$$

$$= -2e + 4e + e + \frac{5e}{6} + e - \frac{5e}{6} = 4e \quad (\approx 10{,}87)$$

(34.) Laske puolisuunnikassäännön avulla $\int_0^2 e^{x^2}\,dx$ tarkkuudella $n=4$.

Jaetaan väli $[0,2]$ neljään yhtäsuureen osaan, joiden pituus $\Delta x = \frac{1}{2}$.

Saadaan jakopisteet $x_0 = 0$, $x_1 = \frac{1}{2}$, $x_2 = 1$, $x_3 = \frac{3}{2}$, $x_4 = 2$

Vastaavat funktion arvot:

$y_0 = f(x_0) = e^{0^2} = 1$

$y_1 = f(x_1) = e^{(\frac{1}{2})^2} = e^{\frac{1}{4}}$

$y_2 = f(x_2) = e^{1^2} = e$

$y_3 = f(x_3) = e^{(\frac{3}{2})^2} = e^{\frac{9}{4}}$

$y_4 = f(x_4) = e^{2^2} = e^4$

$$\int_0^2 e^{x^2}\,dx \approx \Delta x \cdot \left[\frac{1}{2}(y_0 + y_4) + \sum_{i=1}^{3} y_i\right]$$

$$= \frac{1}{2}\left(\frac{1}{2}(1 + e^4) + e^{\frac{1}{4}} + e + e^{\frac{9}{4}}\right)$$

$$= \frac{1}{4} + \frac{e^4}{4} + \frac{e^{1/4}}{2} + \frac{e}{2} + \frac{e^{9/4}}{2}$$

$$\approx 20{,}645$$

(35.) Esitä funktiolle $f(x) = 2x^2 + 2x + 2$ Taylorin sarjakehitelmä

$f(x) = 2x^2 + 2x + 2$, valitaan $a = 0$

$f(x) = 2x^2 + 2x + 2$: $f(0) = 2$

$f'(x) = 4x + 2$ $f'(0) = 2$

$f''(x) = 4$ $f''(0) = 4$

$f^{(k)} = 0$, $k = 3, 4, 5, \ldots$ $f^{(k)}(0) = 0$, $k = 3, 4, 5, \ldots$

$$f(x) = f(0) + f'(0)(x-0) + \frac{f''(0)}{2!}(x-0)^2$$

$$= 2 + 2x + \frac{4}{2!}x^2$$

$$= 2 + 2x + 2x^2$$

$$= 2x^2 + 2x + 2$$

Huom! Toimii myös esim. valinnalla $a=1$.

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(36) a) $a = 1 + 2i$, $b = 1 - 3i$

$\bar{a} = 1 - 2i$

$a + b = 1 + 2i + 1 - 3i = 2 - i$

$a \cdot b = (1+2i) \cdot (1-3i) = 1 - 3i + 2i - 6i^2$ $\quad \| i^2 = -1$

$= 1 - i + 6$

$= 7 - i$

$\frac{a}{b} = \frac{1+2i}{1-3i}^{(1+3i)} = \frac{1 + 2i + 3i + 6i^2}{1^2 - 9i^2} = \frac{-5+5i}{10} = -\frac{1}{2} + \frac{1}{2}i$

$|b| = \sqrt{1^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$

b) $2x^3 - 2x^2 + 18 - 18 = 0$, $x \in \mathbb{C}$

$2(x^3 - x^2 + 9 - 9) = 0$

Mahdolliset ratkaisut: $p: \pm 1, \pm 3, \pm 9$ $\quad \frac{p}{q}: \pm 1, \pm 3, \pm 9$

$q: \pm 1$

Testataan $x = 1$ $\quad 1^3 - 1^2 + 9 - 9 = 0$

$0 = 0$

$\Rightarrow x = 1$ eräs nollakohta

$\Rightarrow x - 1$ eräs tekijä

$$
\begin{array}{r|l}
 & x^2 \quad +9 \\
\hline
x-1 & x^3 - x^2 + 9x - 9 \\
 & -(x^3 - x^2) \\
\hline
 & \quad 0 \quad + 9x - 9 \\
 & \quad -(9x - 9) \\
\hline
 & \qquad\qquad 0
\end{array}
$$

$$\Rightarrow 2(x^3 - x^2 + 9x - 9) = 2(x-1)(x^2+9) = 0$$

$$x^2 + 9 = 0$$

$$x^2 = -9$$

$$x = \pm\sqrt{-9} = \pm\sqrt{9 \cdot i^2} = \pm\sqrt{(3i)^2} = \pm 3i$$

$$\Rightarrow 2x^3 - 2x^2 + 18x - 18 = 2(x-1)(x-3i)(x+3i)$$

Ratkaisut: $x = 1$

$x = 3i$

$x = -3i$.