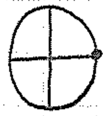
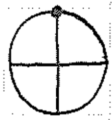
MPTT 2, viikko 17

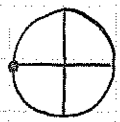
(37.)



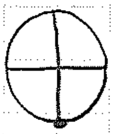
a) $\cos 0 = 1$ b) $\sin 0 = 0$ c) $\tan 0 = \dfrac{\sin 0}{\cos 0} = 0$



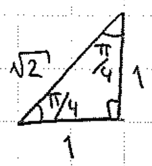
d) $\cos \frac{\pi}{2} = 0$ e) $\sin \frac{\pi}{2} = 1$ f) $\tan\frac{\pi}{2}$ $\not\exists$, koska $\tan\frac{\pi}{2} = \dfrac{\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}}$ ja $\cos\frac{\pi}{2} = 0$



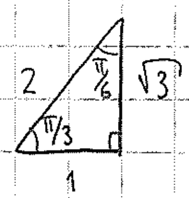
g) $\cos\pi = -1$ h) $\sin\pi = 0$ i) $\tan\pi = \dfrac{\sin\pi}{\cos\pi} = 0$



j) $\cos\frac{3\pi}{2} = 0$ k) $\sin\frac{3\pi}{2} = -1$ l) $\tan\frac{3\pi}{2}$ $\not\exists$



m) $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ n) $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ o) $\tan\frac{\pi}{4} = \dfrac{\cos \pi/4}{\sin \pi/4} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$



p) $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

q) $\sin\frac{\pi}{6} = \frac{1}{2}$ r) $\tan\frac{\pi}{6} = \dfrac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

s) $\cos\frac{\pi}{3} = \frac{1}{2}$ t) $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ u) $\tan\frac{\pi}{3} = \dfrac{\sqrt{3}/2}{1/2} = \sqrt{3}$

(38.) $\sin 2x = \cos x$ $\Big|\ \cos x = \sin(\frac{\pi}{2} - x)$

$\Leftrightarrow \sin 2x = \sin(\frac{\pi}{2} - x)$

$\Leftrightarrow 2x = \frac{\pi}{2} - x + n\,2\pi,\ n \in \mathbb{Z}$ tai $2x = \pi - (\frac{\pi}{2} - x) + n2\pi,\ n \in \mathbb{Z}$

$\Leftrightarrow 3x = \frac{\pi}{2} + n\,2\pi,\ n \in \mathbb{Z}$ tai $2x = \frac{\pi}{2} + x + n2\pi,\ n \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{6} + n \cdot \frac{2\pi}{3},\ n \in \mathbb{Z}$ tai $x = \frac{\pi}{2} + n2\pi,\ n \in \mathbb{Z}$

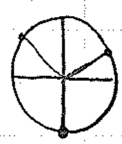
Tarkistetaan, voiko tuloksia yhdistää.

$n=0 \Rightarrow x = \pi/6$ $n=3 \Rightarrow x = 13\pi/6$

$n=1 \Rightarrow x = 5\pi/6$

$n=2 \Rightarrow x = 3\pi/2$

V: $x = \frac{\pi}{6} + n\frac{2\pi}{3}$ tai $x = \frac{\pi}{2} + n2\pi,\ n \in \mathbb{Z}$



(39) a) $\int_0^{\pi/2} \cos 3x \, dx = \frac{1}{3}\int_0^{\pi/2} 3\cos 3x \, dx = \frac{1}{3} \Big/_0^{\pi/2} \sin 3x$

$= \frac{1}{3}(\sin(3\cdot\frac{\pi}{2}) - \sin(3\cdot 0)) = \frac{1}{3}(-1-0) = -\frac{1}{3}$

b) $\int \tan x \, dx = -\int \frac{-\sin x}{\cos x} \, dx = -\ln|\cos x| + C$

c) $\int \sin^2 x \, dx$

$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x$

$\cos 2x = \cos^2 x - \sin^2 x \Rightarrow \cos^2 x = \cos 2x + \sin^2 x$ sij

$\Rightarrow \sin^2 x = 1 - \cos 2x - \sin^2 x$
$2\sin^2 x = 1 - \cos 2x \quad || :2$
$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

Siis $\int \sin^2 x \, dx = \int \frac{1}{2}(1-\cos 2x) \, dx$

$= \frac{1}{2}(\int 1 \, dx - \frac{1}{2}\int 2\cos 2x \, dx) = \frac{1}{2}(x - \frac{1}{2}\sin 2x) + C$

$= \frac{x}{2} - \frac{1}{4}\sin 2x + C$

d) $D \cos 3x = -\sin 3x \cdot 3 = -3\sin 3x$

e) $D\tan 2x = \begin{cases} \frac{1}{\cos^2 2x} \cdot 2 = \frac{2}{\cos^2 2x} \\ (1+\tan^2 2x)\, 2 = 2 + 2\tan^2 2x \end{cases}$

f) $D\sin^2 2x = D(\sin 2x)^2 = 2\sin 2x \cdot \cos 2x \cdot 2$
$= 4\sin 2x \cos 2x$

(40) a) $\int \frac{1}{\sqrt{9-4x^2}}\,dx$

$$= \int \frac{1}{\sqrt{9(1-\frac{4}{9}x^2)}}\,dx$$

$$= \int \frac{1}{3\sqrt{1-(\frac{2}{3}x)^2}}\,dx$$

$$= \int \frac{1}{3} \cdot \frac{1}{\sqrt{1-(\frac{2}{3}x)^2}}\,dx$$

$$= \frac{1}{2} \int \frac{2}{3} \cdot \frac{1}{\sqrt{1-(\frac{2}{3}x)^2}}\,dx$$

$$= \frac{1}{2} \arcsin \frac{2}{3}x + c$$

$\| \int \frac{1}{\sqrt{1-f(x)^2}} \cdot f'(x)\,dx$

$= \arcsin f(x) + c$

b) $\int \frac{2}{4x^2+9}\,dx$

$$= \int \frac{2}{9+4x^2}\,dx$$

$$= \int \frac{2}{9(1+\frac{4}{9}x^2)}\,dx$$

$$= \frac{1}{3} \int \frac{2}{3} \cdot \frac{1}{1+(\frac{2}{3}x)^2}\,dx$$

$$= \frac{1}{3} \arctan \frac{2}{3}x + c$$

$\| \int \frac{1}{1+f(x)^2} \cdot f'(x)\,dx$

$= \arctan f(x) + c$