

1.

a) arvot välillä 0-15

b) $P(X < 5) = 4/8 = \underline{0.50}$

c) $P(5 \leq X < 10) = 3/8 = \underline{0.375}$

d) a on X:n jakauman mediaani, kun $P(X \leq a) = 0.5 \Rightarrow \underline{a = 5}$

e) $E(X)$ on X:n jakauman painopiste, joten $\underline{E(X) > 5}$

f) $P(X = 10) = \underline{0}$, koska jatkuvalla jakaumalla yksittäisen arvon tn. on nolla.

2. $X \sim \text{Tas}[0, 15]$

a) $P(X = 5) = \underline{0}$ (jatkuva jakauma)

b) $P(X \leq 5) = F(5)$
 $= 5/15 \approx \underline{0.3333}$

$$F(x) = \begin{cases} \frac{x-0}{15-0} = x/15, & \text{kun } 0 \leq x \leq 15 \\ 0, & \text{muulloin} \end{cases}$$

c) $P(X \geq 5) = 1 - P(X < 5)$
 $= 1 - P(X \leq 5)$
 $= 1 - F(5) = 1 - 5/15 = 2/3 \approx \underline{0.6667}$

d) $P(5 \leq X \leq 10) = P(X \leq 10) - P(X \leq 5)$
 $= F(10) - F(5)$
 $= 10/15 - 5/15 = 5/15 \approx \underline{0.3333}$

e) $E(X) = \frac{a+b}{2} = \frac{0+15}{2} = \underline{7.5}$

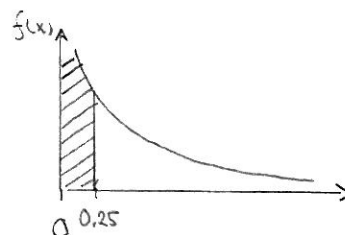
3. $X \sim \text{Exp}(\alpha)$ ja $E(X) = 1/\alpha = 2.5 \Rightarrow \alpha = 0.4$

$\Rightarrow X \sim \text{Exp}(0.4)$

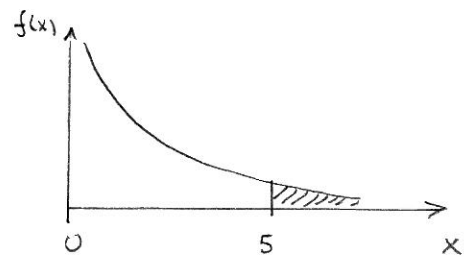
$$\Rightarrow F(x) = P(X \leq x) = \begin{cases} 1 - e^{-0.4x}, & \text{kun } x \geq 0 \\ 0, & \text{kun } x < 0 \end{cases}$$

a)

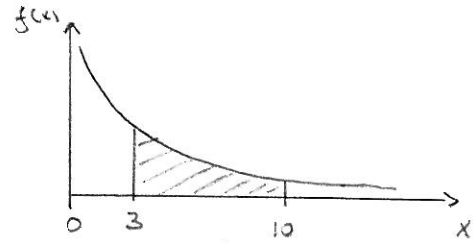
a1) $P(X \leq 15 \text{ sek}) = P(X \leq 1/4 \text{ min})$
 $= F(0.25)$
 $= 1 - e^{-0.4 \cdot 0.25}$
 $= 1 - e^{-0.1}$
 ≈ 0.0952 eli noin 9.5 %



$$\begin{aligned}
 \text{a2) } P(X \geq 5) &= 1 - P(X < 5) \\
 &= 1 - P(X \leq 5) \\
 &= 1 - F(5) \\
 &= 1 - (1 - e^{-0.4 \cdot 5}) \\
 &= e^{-2} \approx 0.1353 \text{ eli noin } \underline{\underline{13.5\%}}
 \end{aligned}$$



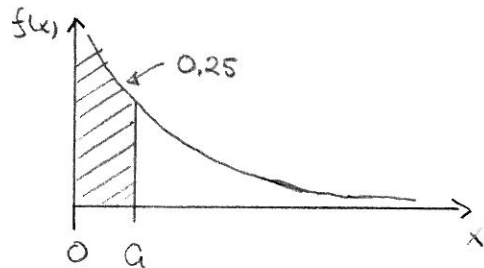
$$\begin{aligned}
 \text{a3) } P(3 \leq X \leq 10) &= P(X \leq 10) - P(X \leq 3) \\
 &= F(10) - F(3) \\
 &= 1 - e^{-0.4 \cdot 10} - (1 - e^{-0.4 \cdot 3}) \\
 &= e^{-1.2} - e^{-4} \approx 0.2829 \text{ eli noin } \underline{\underline{28.3\%}}
 \end{aligned}$$



$$\text{b) } P(X \geq 10 + 5 \mid X \geq 10) = P(X \geq 5) \stackrel{\text{a2)}}{\approx} \underline{\underline{0.1353}} \text{ (eksponenttijakauman unoh-}$$

vaisuusominaisuus)

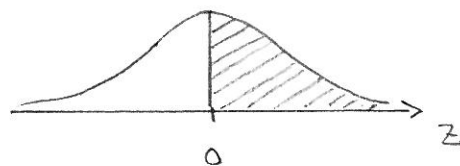
$$\begin{aligned}
 \text{c) } Q_1 = a &\Leftrightarrow P(X \leq a) = 0.25 \\
 &\Leftrightarrow F(a) = 0.25 \\
 &\Leftrightarrow 1 - e^{-0.4a} = 0.25 \\
 &\Leftrightarrow e^{-0.4a} = 0.75 \quad |\ln| \\
 &\Leftrightarrow -0.4a = \ln(0.75) \\
 &\Rightarrow a = -\frac{\ln(0.75)}{0.4} \approx \underline{\underline{0.7192}}
 \end{aligned}$$



$$D^2(X) = 1/\alpha^2 = 1/0.4^2 = \underline{\underline{6.25}}$$

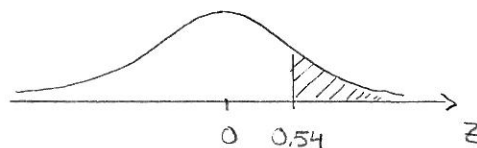
④ $Z \sim N(0,1)$

$$\text{a) } P(Z > 0) = \underline{\underline{0.5000}}$$

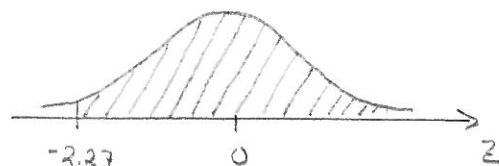


$$\text{b) } P(Z \geq 0) = \underline{\underline{0.5000}}$$

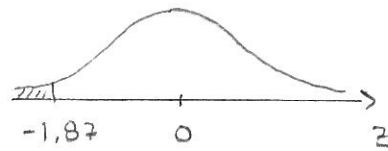
$$\text{c) } P(Z > 0.54) = \underline{\underline{0.2946}}$$



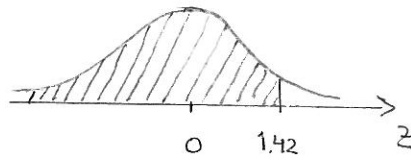
$$\begin{aligned}
 \text{d) } P(Z > -2.27) &= 1 - P(Z \leq -2.27) \\
 &= 1 - P(Z \geq 2.27) \\
 &= 1 - 0.0116 \\
 &= \underline{\underline{0.9884}}
 \end{aligned}$$



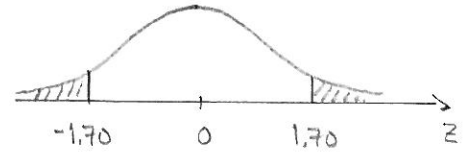
$$e) P(Z < -1.87) = P(X > 1.87) \\ = \underline{\underline{0.0307}}$$



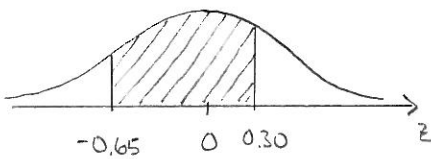
$$f) P(Z \leq 1.42) = 1 - P(Z > 1.42) \\ = 1 - 0.0778 \\ = \underline{\underline{0.9222}}$$



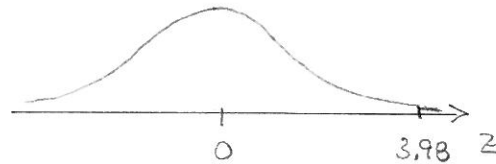
$$g) P(|Z| > 1.70) = P(Z < -1.70 \text{ or } Z > 1.70) \\ = P(Z < -1.70) + P(Z > 1.70) \\ = 2 \cdot P(Z > 1.70) \\ = 2 \cdot 0.0446 \\ = \underline{\underline{0.0892}}$$



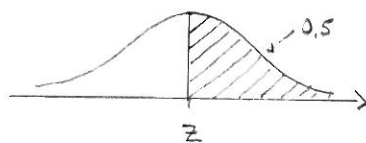
$$h) P(-0.65 \leq Z \leq 0.30) = P(Z \geq -0.65) - P(Z > 0.30) \\ = 1 - P(Z < -0.65) - P(Z > 0.30) \\ = 1 - P(Z > 0.65) - P(Z > 0.30) \\ = 1 - 0.2578 - 0.3821 \\ = \underline{\underline{0.3601}}$$



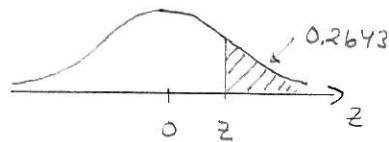
$$i) P(Z > 3.98) = \underline{\underline{0.0000}}$$



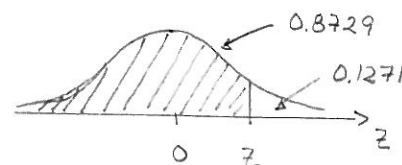
5. a) $P(Z \geq z) = 0.5$
 $\Rightarrow z = \underline{\underline{0.00}}$



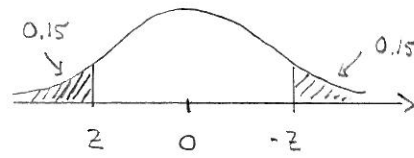
b) $P(Z \geq z) = 0.2643$
 $\Rightarrow z = \underline{\underline{0.63}}$



c) $P(Z \leq z) = 0.8729$
 $\Leftrightarrow P(Z > z) = 1 - 0.8729 = 0.1271$
 $\Rightarrow z \approx \underline{\underline{1.14}}$

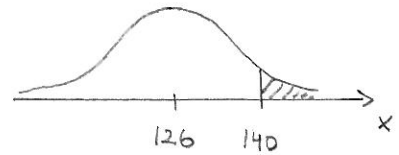


d) $P(Z \leq z) = 0.1500$
 $\Leftrightarrow P(Z \geq -z) = 0.1500$
 $\Rightarrow -z \approx 1.04 \Rightarrow z = \underline{\underline{-1.04}}$

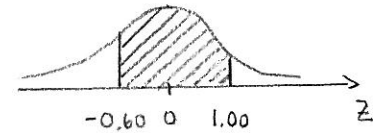
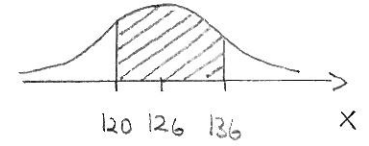


6. Merk. $X = \text{SRT-pistemäärä}$, $X \sim N(126, 10^2)$

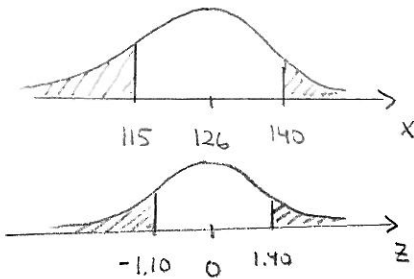
$$\begin{aligned} \text{a) } P(X > 140) &= P\left(\underbrace{\frac{X-126}{10}}_{Z \sim N(0,1)} > \frac{140-126}{10}\right) \\ &= P(Z > 1.40) = \underline{\underline{0.0808}} \end{aligned}$$



$$\begin{aligned} \text{b) } P(120 \leq X \leq 136) &= P\left(\frac{120-126}{10} \leq \underbrace{\frac{X-126}{10}}_{Z \sim N(0,1)} \leq \frac{136-126}{10}\right) \\ &= P(-0.60 \leq Z \leq 1.00) \\ &= P(Z > -0.60) - P(Z > 1.00) \\ &= 1 - P(Z > 0.60) - P(Z > 1.00) \\ &= 1 - 0.2743 - 0.1587 \\ &= \underline{\underline{0.5670}} \end{aligned}$$



$$\begin{aligned} \text{c) } P(X < 115 \text{ tai } X > 140) &= P(X < 115) + P(X > 140) \\ &= P\left(\underbrace{\frac{X-126}{10}}_{Z \sim N(0,1)} < \frac{115-126}{10}\right) + P\left(\underbrace{\frac{X-126}{10}}_{Z \sim N(0,1)} < \frac{140-126}{10}\right) \\ &= P(Z < -1.10) + P(Z > 1.40) \\ &= P(Z > 1.10) + P(Z > 1.40) \\ &= 0.1357 + 0.0808 = \underline{\underline{0.2165}} \end{aligned}$$

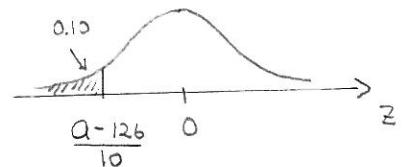
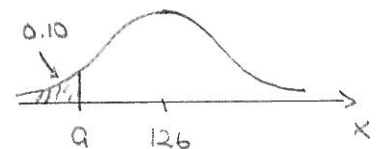


7. $X \sim N(126, 10^2)$

$$\text{a) } P(X \leq a) = 0.10 \Leftrightarrow P\left(\frac{X-126}{10} \leq \frac{a-126}{10}\right) = 0.10$$

$$\begin{aligned} N(0,1) \text{-jakauman taulukko: } P(Z \geq 1.28) &\approx 0.10 \\ \Leftrightarrow P(Z \leq -1.28) &\approx 0.10 \end{aligned}$$

$$\Rightarrow \frac{a-126}{10} \approx -1.28 \Rightarrow a \approx -1.28 \cdot 10 + 126 = \underline{\underline{113.20}}$$



$$\text{b) } P(X \leq a) = 0.75 \Leftrightarrow P(X > a) = 0.25$$

$$\Leftrightarrow P\left(\underbrace{\frac{X-126}{10}}_{Z \sim N(0,1)} > \frac{a-126}{10}\right) = 0.25$$

$$N(0,1) \text{-jakauman taulukko: } P(Z \geq 0.67) \approx 0.25$$

$$\Rightarrow \frac{a-126}{10} \approx 0.67 \Rightarrow a \approx 0.67 \cdot 10 + 126 = \underline{\underline{132.70}}$$

