(1.) Harjoitus 10, tehtävä 6a: tn., että satunnaisesti valitun henkilön SRT-pistemäärä on yli 140 pistettä on 0,0808

Merk. Y = SRT-pistemäärän "yli 140" omaavien lkm kahdeksan testatun joukossa

$Y \sim Bin(8, 0.0808)$

$$
\begin{aligned}
P(Y \leq 1) &= P(Y=0) + P(Y=1) \\
&= \binom{8}{0} \cdot 0.0808^0 \cdot (1-0.0808)^{8-0} + \binom{8}{1} \cdot 0.0808^1 \cdot (1-0.0808)^{8-1} \\
&\approx 0.5097 + 0.3584 \\
&= \underline{\underline{0.8681}}
\end{aligned}
$$

(2.) $X \sim N(8, 3^2)$ ja $Y \sim N(11, 4^2)$, $X \perp\!\!\!\perp Y$

a) a1) $P(X<0) = P(\underbrace{\frac{X-8}{3}}_{=Z \sim N(0,1)} < \frac{0-8}{3}) = P(Z < -2.67)$

$= P(Z > 2.67) = \underline{\underline{0.0038}}$

a2) $P(Y<0) = P(\underbrace{\frac{Y-11}{4}}_{=Z \sim N(0,1)} < \frac{0-11}{4}) = P(Z < -2.75)$

$= P(Z > 2.75) = \underline{\underline{0.0030}}$

b) $P(X>6) = P(\underbrace{\frac{X-8}{3}}_{=Z \sim N(0,1)} > \frac{6-8}{3}) = P(Z > -0.67)$

$= 1 - P(Z \leq -0.67)$

$= 1 - P(Z \geq 0.67) = 1 - 0.2514 = \underline{0.7486}$

$P(Y>6) = P(\underbrace{\frac{Y-11}{4}}_{=Z \sim N(0,1)} > \frac{6-11}{4}) = P(Z > -1.25)$

$= 1 - P(Z \leq -1.25)$

$= 1 - P(Z \geq 1.25) = 1 - 0.1057 = \underline{0.8943}$

$\Rightarrow$ <u>vaihtoehto B</u>

c) Merk. $S = X - Y$ $E(S) = E(X-Y) = E(X) - E(Y)$

$= 8 - 11 = -3$

$$D^2(S) = D^2(X - Y) = D^2(X) + D^2(Y)$$
$$= 3^2 + 4^2 = 9 + 16 = 25$$

$$\Rightarrow \quad S \sim N(-3, 5^2)$$

$$P(S > 0) = P\Big(\underbrace{\frac{S - -3}{5}}_{= Z \sim N(0,1)} > \frac{0 - -3}{5}\Big) = P(Z > 0.60)$$
$$= \underline{\underline{0.2743}}$$

(3.) Merk. X_i = asiakkaan i kohdalla aiheutuvan tappion määrä senteissä

$X_i \sim \mathrm{Tas}[-2.5, 2.5]$, kun $i = 1, \ldots, 1000$

$$\Rightarrow \quad \mu = E(X_i) = \frac{a+b}{2} = \frac{-2.5 + 2.5}{2} = 0 \quad \text{ja}$$
$$\sigma^2 = D^2(X_i) = \frac{(b-a)^2}{12} = \frac{(2.5 - -2.5)^2}{12} = \frac{25}{12} \quad , \text{kun } i = 1, \ldots, 1000$$

Merk. $S = X_1 + X_2 + \ldots + X_{1000}$

$$E(S) = E(X_1 + X_2 + \ldots + X_{1000}) = E(X_1) + E(X_2) + \ldots + E(X_{1000})$$
$$= 0 + 0 + \ldots + 0 = 0$$

$$D^2(S) = D^2(X_1 + X_2 + \ldots + X_{1000}) = D^2(X_1) + D^2(X_2) + \ldots + D^2(X_{1000})$$
$$= 25/12 + 25/12 + \ldots + 25/12$$
$$= \frac{25000}{12} \approx \sqrt{2083.33}^2$$

$$\Rightarrow \quad S \sim N(0, \sqrt{2083.33}^2) \quad \text{likimain} \quad (\text{keskeinen raja-arvolause})$$

$$\underset{\substack{\uparrow \\ \text{1 euro} \\ \text{sentteinä}}}{P(S > 100)} = P\Big(\underbrace{\frac{S - 0}{\sqrt{2083.33}}}_{= Z \sim N(0,1)} > \frac{100 - 0}{\sqrt{2083.33}}\Big) = P(Z > 2.19)$$
$$= \underline{\underline{0.0143}}$$

(4.) Merk. X = palautettujen tuotteiden lkm 2500 tilatun tuotteen joukossa

$X \sim \mathrm{Bin}(2500, 0.11)$

Koska $np = 2500 \cdot 0.11 = 275 > 5$ ja
$n(1-p) = 2500 \cdot (1 - 0.11) = 2225 > 5$

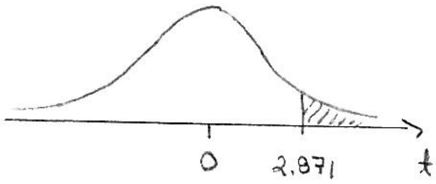
$$\Rightarrow \quad \mathrm{Bin}(2500, 0.11) \approx N(\overset{n \cdot p}{275}, \overset{n \cdot p \cdot (1-p)}{2500 \cdot 0.11 \cdot (1 - 0.11)}) = N(275, \sqrt{244.75}^2)$$

jatkuvuuskorjaus

$$P(X < 300.5) = P\Big(\underbrace{\frac{X-275}{\sqrt{244.75}}}_{=Z\sim N(0,1)} < \frac{300.5-275}{\sqrt{244.75}}\Big) = P(Z < 1.63)$$
$$= 1 - P(Z \geq 1.63)$$
$$= 1 - 0.0516 = \underline{\underline{0.9484}}$$

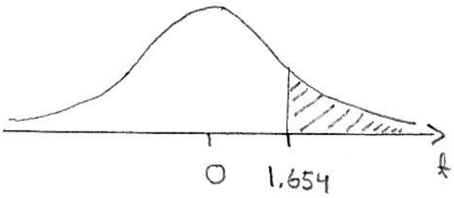
5. $T \sim t(15)$

a) a1)



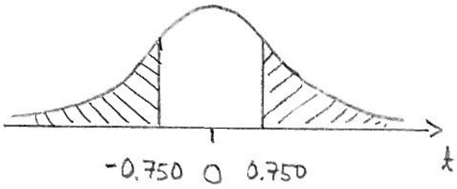
$$\underline{\underline{0.005 < P(T \geq 2.871) < 0.01}}$$

a2)



$$\underline{\underline{0.05 < P(T \geq 1.654) < 0.10}}$$

a3)

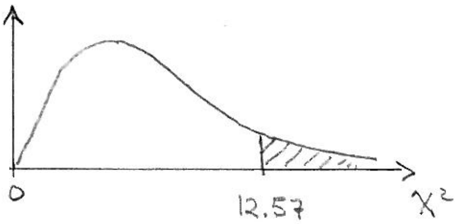


$$P(|T| \geq 0.750) = P(T \leq -0.750 \text{ tai } T \geq 0.750)$$
$$= P(T \leq -0.750) + P(T \geq 0.750)$$
$$= 2P(T \geq 0.750)$$

$$\Rightarrow 2 \cdot 0.2 < P(|T| \geq 0.750) < 0.3 \cdot 2$$
$$\Leftrightarrow \underline{\underline{0.4 < P(|T| \geq 0.750) < 0.6}}$$

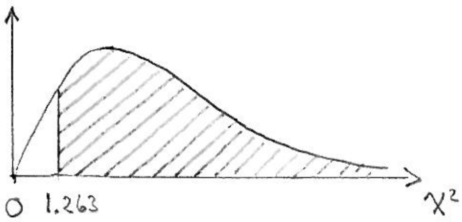
b) $\chi^2 \sim \chi^2(3)$

b1)



$$\underline{\underline{0.001 < P(\chi^2 \geq 12.57) < 0.01}}$$

b2)



$$\underline{\underline{0.50 < P(\chi^2 \geq 1.263) < 0.80}}$$

6. $X_i \sim N(126, 10^2) \Rightarrow \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim N(126, 10^2/n)$

a) $\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i \sim N(126, 10^2/10) = N(126, \sqrt{10}^2)$

$$P(\bar{X} \leq 125) = P\Big(\underbrace{\frac{\bar{X}-126}{\sqrt{10}}}_{=Z\sim N(0,1)} \leq \frac{125-126}{\sqrt{10}}\Big) = P(Z \leq -0.32) = P(Z \geq 0.32)$$
$$= \underline{\underline{0.3745}}$$

b) $\bar{X} = \frac{1}{25}\sum_{i=1}^{25} X_i \sim N(126, 10^2/25) = N(126, 2^2)$

$$P(\bar{X} \leq 125) = P(\underbrace{\frac{\bar{X}-126}{2}}_{=Z\sim N(0,1)} \leq \frac{125-126}{2}) = P(Z \leq -0{,}50)$$

$$= P(Z \geq 0{,}50) = \underline{\underline{0{,}3085}}$$

c) $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim N(126, 10^2/n)$

$$P(\bar{X} \leq 125) = 0{,}1587 \iff P(\underbrace{\frac{\bar{X}-126}{10/\sqrt{n}}}_{=Z\sim N(0,1)} \leq \frac{125-126}{10/\sqrt{n}}) = 0{,}1587$$

N(0,1)-jakauman taulukko: $P(Z \geq 1{,}00) = 0{,}1587$

$$\iff P(Z \leq -1{,}00) = 0{,}1587$$

$$\Rightarrow \frac{125-126}{10/\sqrt{n}} = -1{,}00 \iff \frac{\sqrt{n}\cdot(125-126)}{10} = -1{,}00$$

$$\iff -\sqrt{n} = -10 \quad |()^2$$

$$\Rightarrow \underline{\underline{n = 100}}$$

7. a) Kun $X \sim Tas[0,10]$, niin $\mu = E(X) = \frac{a+b}{2} = \frac{0+10}{2} = 5$

$$\sigma^2 = D^2(X) = \frac{(b-a)^2}{12} = \frac{(10-0)^2}{12} = \frac{100}{12}$$

a1) $\Rightarrow \bar{X} = \frac{1}{100}\sum_{i=1}^{100} X_i \sim N(5, \frac{100/12}{100}) = \underline{\underline{N(5, \sqrt{1/12}^2)}}$ likimain

a2) $P(\bar{X} < 5{,}2) = P(\underbrace{\frac{\bar{X}-5}{\sqrt{1/12}}}_{=Z\sim N(0,1)} < \frac{5{,}2-5}{\sqrt{1/12}}) = P(Z < 0{,}69) = 1 - P(Z \geq 0{,}69)$

$$= 1 - 0.2451 = \underline{\underline{0{,}7549}}$$

b) Kun $X \sim Exp(0.2)$, niin $\mu = E(X) = 1/\alpha = 1/0.2 = 5$

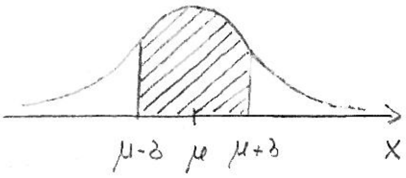
$$\sigma^2 = D^2(X) = 1/\alpha^2 = 1/0.2^2 = 25$$

b1) $\Rightarrow \bar{X} = \frac{1}{50}\sum_{i=1}^{50} X_i \sim N(5, 25/50) = \underline{\underline{N(5, \sqrt{0.5}^2)}}$ likimain

b2) $P(\bar{X} > 5{,}2) = P(\underbrace{\frac{\bar{X}-5}{\sqrt{0{,}5}}}_{=Z\sim N(0,1)} > \frac{5{,}2-5}{\sqrt{0{,}5}}) = P(Z > 0.28) = \underline{\underline{0{,}3897}}$

(8.) $X \sim N(\mu, \sigma^2)$

a) $P(\mu-\sigma \leq X \leq \mu+\sigma) = P\left(\frac{\mu-\sigma-\mu}{\sigma} \leq \underbrace{\frac{X-\mu}{\sigma}}_{= Z \sim N(0,1)} \leq \frac{\mu+\sigma-\mu}{\sigma}\right)$



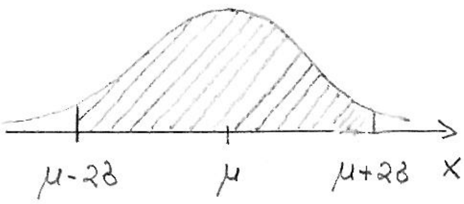
$= P(-1.00 \leq Z \leq 1.00)$

$= 1 - 2P(Z \geq 1.00)$

$= 1 - 2 \cdot 0.1587$

$= \underline{\underline{0.6826}}$

b) $P(\mu-2\sigma \leq X \leq \mu+2\sigma) = P\left(\frac{\mu-2\sigma-\mu}{\sigma} \leq \underbrace{\frac{X-\mu}{\sigma}}_{= Z \sim N(0,1)} \leq \frac{\mu+2\sigma-\mu}{\sigma}\right)$



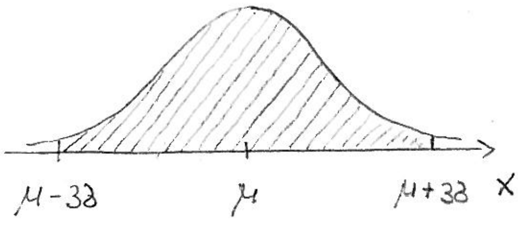
$= P(-2.00 \leq Z \leq 2.00)$

$= 1 - 2P(Z \geq 2.00)$

$= 1 - 2 \cdot 0.0228$

$= \underline{\underline{0.9544}}$

c) $P(\mu-3\sigma \leq X \leq \mu+3\sigma) = P\left(\frac{\mu-3\sigma-\mu}{\sigma} \leq \underbrace{\frac{X-\mu}{\sigma}}_{= Z \sim N(0,1)} \leq \frac{\mu+3\sigma-\mu}{\sigma}\right)$



$= P(-3.00 \leq Z \leq 3.00)$

$= 1 - 2P(Z \geq 3.00)$

$= 1 - 2 \cdot 0.0014$

$= \underline{\underline{0.9972}}$