Matematiikan perusteet taloustieteilijöille II

Harjoitusten malliratkaisut, viikko 12, kevät 2014

24. a) Koska $D(x^2+2x+2) = 2x+2 = 2(x+1)$, niin

$$\begin{aligned}\int \frac{x+1}{\sqrt[3]{x^2+2x+2}}dx &= \int (x+1)(x^2+2x+2)^{-\frac{1}{3}}dx \\ &= \frac{1}{2}\int 2(x+1)(x^2+2x+2)^{-\frac{1}{3}}dx \\ &= \frac{1}{2}\cdot\frac{(x^2+2x+2)^{\frac{2}{3}}}{\frac{2}{3}} + c \\ &= \frac{3}{4}(x^2+2x+2)^{\frac{2}{3}} + c.\end{aligned}$$

b) Koska $Dx^2 = 2x$, niin

$$\int xe^{x^2}dx = \frac{1}{2}\int 2xe^{x^2}dx = \frac{1}{2}e^{x^2} + c.$$

c) Koska $D\ln(x^2) = \frac{1}{x^2}\cdot 2x = \frac{2}{x}$, niin

$$\begin{aligned}\int \frac{x}{x^2\ln(x^2)}dx &= \int \frac{1}{x\ln(x^2)}dx = \frac{1}{2}\int \frac{2}{x\ln(x^2)}dx \\ &= \frac{1}{2}\int \frac{\frac{2}{x}}{\ln(x^2)}dx = \frac{1}{2}\ln|\ln(x^2)| + c.\end{aligned}$$

d) Koska $D(x^2+1) = 2x$, niin

$$\int 2^{x^2+1}xdx = \frac{1}{2}\int 2x\cdot 2^{x^2+1}dx = \frac{1}{2}\cdot\frac{2^{x^2+1}}{\ln 2} + c = \frac{2^{x^2}}{\ln 2} + c.$$

e) Koska $D(2x^2+4x+5) = 4x+4 = 4(x+1)$, niin

$$\int \frac{x+1}{2x^2+4x+5}dx = \frac{1}{4}\int \frac{4(x+1)}{2x^2+4x+5}dx = \frac{1}{4}\ln|2x^2+4x+5| + c.$$

25. a)

$$\int x^2|x|dx = \begin{cases} \int x^2 \cdot x dx, \text{ kun } x \geq 0 \\ \int x^2 \cdot (-x) dx, \text{ kun } x < 0 \end{cases}$$
$$= \begin{cases} \int x^3 dx, \text{ kun } x \geq 0 \\ \int -x^3 dx, \text{ kun } x < 0 \end{cases}$$
$$= \begin{cases} \frac{1}{4}x^4 + c, \text{ kun } x \geq 0 \\ -\frac{1}{4}x^4 + c, \text{ kun } x < 0 \end{cases}$$
$$= \frac{1}{4}x^3|x| + c$$

b)

$$\int \frac{dx}{1+e^x} = \int \frac{1+e^x-e^x}{1+e^x}dx = \int \left(1 - \frac{e^x}{1+e^x}\right) dx$$
$$= x - \ln|\underbrace{1+e^x}_{>0}| + c = x - \ln(1+e^x) + c$$

c)

$$\int \frac{\ln x}{x}dx = \int (\ln x)^1 \cdot \frac{1}{x}dx = \frac{1}{2}(\ln x)^2 + c$$

26. a)

$$\int (x^2 - \sqrt{x} + 2)dx = \int (x^2 - x^{\frac{1}{2}} + 2)dx = \frac{1}{3}x^3 - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2x + c$$
$$= \frac{1}{3}x^3 - \frac{2}{3}x^{\frac{3}{2}} + 2x + c = \frac{1}{3}x^3 - \frac{2}{3}x\sqrt{x} + 2x + c$$

b)

$$\int \sqrt{2+5x}dx = \int (2+5x)^{\frac{1}{2}}dx = \frac{1}{5}\int 5(2+5x)^{\frac{1}{2}}dx$$
$$= \frac{1}{5} \cdot \frac{(2+5x)^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{15}(2+5x)^{\frac{3}{2}} + c$$

c)

$$\int \frac{dx}{(3x+2)^2} = \int (3x+2)^{-2}dx = \frac{1}{3}\int 3(3x+2)^{-2}dx$$
$$= \frac{1}{3}\cdot\frac{(3x+2)^{-1}}{-1} + c = -\frac{1}{3(3x+2)} + c = -\frac{1}{9x+6} + c$$

d)

$$\int \frac{x^3-1}{x-1}dx = \int \frac{(x-1)(x^2+x+1)}{x-1}dx = \int (x^2+x+1)dx$$
$$= \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + c$$

27. a) Valitaan $f'(x) = e^{-x}$ ja $g(x) = x$. Tällöin $f(x) = -e^{-x}$ ja $g'(x) = 1$. Nyt osittaisintegroinnilla saadaan

$$\int xe^{-x}dx = x\cdot(-e^{-x}) - \int(-e^{-x}\cdot 1)dx = -xe^{-x} - \int -e^{-x}dx$$
$$= -xe^{-x} - e^{-x} + c = -e^{-x}(x+1) + c.$$

b) Valitaan $f'(x) = x^7$ ja $g(x) = \ln x$. Tällöin $f(x) = \frac{1}{8}x^8$ ja $g'(x) = \frac{1}{x}$. Nyt osittaisintegroinnilla saadaan

$$\int x^7 \ln x dx = \frac{1}{8}x^8\cdot\ln x - \int\left(\frac{1}{8}x^8\cdot\frac{1}{x}\right)dx = \frac{1}{8}x^8\ln x - \int\frac{1}{8}x^7dx$$
$$= \frac{1}{8}x^8\ln x - \frac{1}{8}\cdot\frac{1}{8}x^8 + c = \frac{1}{8}x^8(\ln x - \frac{1}{8}) + c.$$