

MPTT a, viikko 5

$$\textcircled{4} \text{ c) } \begin{vmatrix} -2 & -1 & 4 \\ 6 & -3 & -2 \\ 4 & 1 & 2 \end{vmatrix} \begin{matrix} \swarrow \\ \searrow \\ \swarrow \end{matrix} \begin{matrix} (-2) \\ (-1) \\ (-2) \end{matrix} = \begin{vmatrix} -10 & -3 & 0 \\ 10 & -2 & 0 \\ 4 & 1 & 2 \end{vmatrix} \begin{matrix} \downarrow \\ \downarrow \\ \downarrow \end{matrix} = 0 + 0 + a \cdot (-1)^{3+3} \cdot \begin{vmatrix} -10 & -3 \\ 10 & -2 \end{vmatrix}$$

$$= 2 \cdot (-10 \cdot (-2) - (-3) \cdot 10) = 2 \cdot 50 = 100$$

$$\text{d) } \begin{vmatrix} 3 & -2 & -5 & 4 \\ -5 & 2 & 8 & -5 \\ -2 & 4 & 7 & -3 \\ 2 & -3 & -5 & 8 \end{vmatrix} \begin{matrix} \downarrow \\ \downarrow \\ \downarrow \\ \downarrow \end{matrix} \begin{matrix} (-1) \\ (-2) \\ (-1) \\ (-1) \end{matrix} = \begin{vmatrix} 3 & -2 & -5 & 4 \\ -2 & 0 & 3 & -1 \\ 4 & 0 & -3 & 5 \\ 2 & -3 & -5 & 8 \end{vmatrix} \begin{matrix} \swarrow \\ \swarrow \\ \swarrow \\ \swarrow \end{matrix} \begin{matrix} (-1) \\ (-1) \\ (-1) \\ (-1) \end{matrix} = \begin{vmatrix} 1 & 1 & 0 & -4 \\ -2 & 0 & 3 & -1 \\ 4 & 0 & -3 & 5 \\ 2 & -3 & -5 & 8 \end{vmatrix} \begin{matrix} \downarrow \\ \downarrow \\ \downarrow \\ \downarrow \end{matrix} \begin{matrix} (-1) \\ (-1) \\ (-1) \\ (-1) \end{matrix}$$

$$= \begin{vmatrix} 1 & 1 & 0 & -4 \\ -2 & 0 & 3 & -1 \\ 4 & 0 & -3 & 5 \\ 5 & 0 & -5 & -4 \end{vmatrix} \begin{matrix} \downarrow \\ \downarrow \\ \downarrow \\ \downarrow \end{matrix} \begin{matrix} (-1) \\ (-1) \\ (-1) \\ (-1) \end{matrix} = 1 \cdot (-1)^{1+2} \cdot \begin{vmatrix} -2 & 3 & -1 \\ 4 & -3 & 5 \\ 5 & -5 & -4 \end{vmatrix} \begin{matrix} \downarrow \\ \downarrow \\ \downarrow \end{matrix} \begin{matrix} (-1) \\ (-1) \\ (-1) \end{matrix}$$

$$= -1 \cdot \begin{vmatrix} -2 & 3 & -1 \\ -6 & 12 & 0 \\ 13 & -17 & 0 \end{vmatrix} \begin{matrix} \downarrow \\ \downarrow \\ \downarrow \end{matrix} \begin{matrix} (-1) \\ (-1) \\ (-1) \end{matrix} = -1 \cdot (-1) \cdot (-1)^{1+3} \cdot \begin{vmatrix} -6 & 12 \\ 13 & -17 \end{vmatrix}$$

$$= -6 \cdot (-17) - 12 \cdot 13 = 102 - 156 = -54$$

$$\text{e) } \begin{vmatrix} 1 & 1 & 2 & 3 \\ 2 & 2 & -1 & 0 \\ 3 & 2 & 1 & 1 \\ 3 & 2 & 6 & 7 \end{vmatrix} \begin{matrix} \downarrow \\ \downarrow \\ \downarrow \\ \downarrow \end{matrix} \begin{matrix} (-2) \\ (-3) \\ (-3) \\ (-3) \end{matrix} = \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & 0 & -5 & -6 \\ 0 & -1 & -5 & -8 \\ 0 & -1 & 0 & -2 \end{vmatrix} \begin{matrix} \downarrow \\ \downarrow \\ \downarrow \\ \downarrow \end{matrix} \begin{matrix} (-1) \\ (-1) \\ (-1) \\ (-1) \end{matrix}$$

$$= \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & 0 & -5 & -6 \\ 0 & -1 & 0 & -2 \\ 0 & -1 & 0 & -2 \end{vmatrix} \begin{matrix} \downarrow \\ \downarrow \\ \downarrow \\ \downarrow \end{matrix} \begin{matrix} (-1) \\ (-1) \\ (-1) \\ (-1) \end{matrix} \rightarrow \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & 0 & -5 & -6 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{vmatrix} = 0$$

Kaksi samaa vaakariviä, jolloin saadaan yksi nollarivi. Kehitetään nollarivin mukaan, jolloin determinantti on nolla.

$$5. \quad A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

$$\det A = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 2 \cdot 3 - 1 \cdot 5 = 1 \neq 0 \Rightarrow A^{-1} \exists.$$

$$b) \quad K = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad A_{ij} = (-1)^{i+j} |M_{ij}|$$

$$A_{11} = (-1)^{1+1} \cdot 3 = 3$$

$$A_{12} = (-1)^{1+2} \cdot 1 = -1$$

$$A_{21} = (-1)^{2+1} \cdot 5 = -5$$

$$A_{22} = (-1)^{2+2} \cdot 2 = 2$$

$$\Rightarrow K = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$$

$$\text{Nyt} \quad A^{-1} = \frac{1}{\det A} K^T = \frac{1}{1} \cdot \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

$$c) \quad (A : I) = \left( \begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array} \right) \xrightarrow{(-1)} \sim \left( \begin{array}{cc|cc} 1 & 2 & 1 & -1 \\ 1 & 3 & 0 & 1 \end{array} \right) \xrightarrow{(-1)}$$

$$\sim \left( \begin{array}{cc|cc} 1 & 2 & 1 & -1 \\ 0 & 1 & -1 & 2 \end{array} \right) \xrightarrow{(-2)} \sim \left( \begin{array}{cc|cc} 1 & 0 & 3 & -5 \\ 0 & 1 & -1 & 2 \end{array} \right) = (I : A^{-1})$$

$$\text{Eli} \quad A^{-1} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

$$6. \quad A = \begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix}$$

$$\det A = \begin{vmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{vmatrix} \xrightarrow{2} = \begin{vmatrix} 1 & 0 & 0 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{vmatrix}$$

$$= -1 \cdot (-1)^{3+2} \cdot \begin{vmatrix} 1 & 0 \\ -4 & 1 \end{vmatrix} = 1 - 0 \cdot (-4) = 1 \neq 0 \Rightarrow A^{-1} \exists$$



$$a) A_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 1 \\ -1 & -1 \end{vmatrix} = 0 \cdot (-1) - 1 \cdot (-1) = 1$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} -4 & 1 \\ 6 & -1 \end{vmatrix} = -(-4 \cdot (-1) - 1 \cdot 6) = 2$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} -4 & 0 \\ 6 & -1 \end{vmatrix} = -4 \cdot (-1) - 0 \cdot 6 = 4$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 2 \\ -1 & -1 \end{vmatrix} = -(2 \cdot (-1) - 2 \cdot (-1)) = 0$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} -11 & 2 \\ 6 & -1 \end{vmatrix} = -11 \cdot (-1) - 2 \cdot 6 = -1$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} -11 & 2 \\ 6 & -1 \end{vmatrix} = -(-11 \cdot (-1) - 2 \cdot 6) = 1$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 2 \\ 0 & 1 \end{vmatrix} = 2 \cdot 1 - 2 \cdot 0 = 2$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} -11 & 2 \\ -4 & 1 \end{vmatrix} = -(-11 \cdot 1 - 2 \cdot (-4)) = 3$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} -11 & 2 \\ -4 & 0 \end{vmatrix} = -11 \cdot 0 - 2 \cdot (-4) = 8$$

$$\Rightarrow K = \begin{pmatrix} 1 & 2 & 4 \\ 0 & -1 & 1 \\ 2 & 3 & 8 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{\det A} K^T = \frac{1}{\det A} \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$$

$$b) (A|I) = \left( \begin{array}{ccc|ccc} -11 & 2 & 2 & 1 & 0 & 0 \\ -4 & 0 & 1 & 0 & 1 & 0 \\ 6 & -1 & -1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{2} \sim \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ -4 & 0 & 1 & 0 & 1 & 0 \\ 6 & -1 & -1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{2 \cdot 4 \\ (-6)}} (-6)$$

$$\sim \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 & 1 & 8 \\ 0 & -1 & -1 & -6 & 0 & -11 \end{array} \right) \xrightarrow{2} \sim \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & -1 & -1 & -6 & 0 & -11 \\ 0 & 0 & 1 & 4 & 1 & 8 \end{array} \right) \xrightarrow{1}$$

$$\sim \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & -1 & 0 & -2 & 1 & -3 \\ 0 & 0 & 1 & 4 & 1 & 8 \end{array} \right) \cdot (-1) \quad \sim \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 2 & -1 & 3 \\ 0 & 0 & 1 & 4 & 1 & 8 \end{array} \right) = (I|A^{-1})$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$$

Tarkistus:

$$A \cdot A^{-1} = \begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} -11 \cdot 1 + 2 \cdot 2 + 2 \cdot 4 & -11 \cdot 0 + 2 \cdot (-1) + 2 \cdot 1 & -11 \cdot 2 + 2 \cdot 3 + 2 \cdot 8 \\ -4 \cdot 1 + 0 \cdot 2 + 1 \cdot 4 & -4 \cdot 0 + 0 \cdot (-1) + 1 \cdot 1 & -4 \cdot 2 + 0 \cdot 3 + 1 \cdot 8 \\ 6 \cdot 1 + (-1) \cdot 2 + (-1) \cdot 4 & 6 \cdot 0 + (-1) \cdot (-1) + (-1) \cdot 1 & 6 \cdot 2 - 1 \cdot 3 - 1 \cdot 8 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{OK.}$$