MPTT 2, viikko 6

(7.) $\begin{cases} -11x + 2y + 2z = 1 \\ -4x + z = 2 \\ 6x - y - z = 3 \end{cases}$ muuttujien lkm = yhtälöiden lkm

a) $\begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \Leftrightarrow A\bar{x} = \bar{c}$

Tehtävä 6 $\Rightarrow \det A = 1 \neq 0 \Rightarrow A^{-1}\ \exists$
$\Rightarrow$ yhtälöryhmällä on yksikäsitteinen ratkaisu $\bar{x} = A^{-1}\bar{c}$

Tehtävä 6 $\Rightarrow A^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$

$$\bar{x} = A^{-1}\bar{c} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\bar{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \cdot 1 + 0 \cdot 2 + 2 \cdot 3 \\ 2 \cdot 1 + (-1) \cdot 2 + 3 \cdot 3 \\ 4 \cdot 1 + 1 \cdot 2 + 8 \cdot 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \\ 30 \end{pmatrix} \Leftrightarrow \begin{cases} x = 7 \\ y = 9 \\ z = 30 \end{cases}$$

b) Voidaan käyttää Cramerin sääntöä, koska $\det A = 1 \neq 0$ ja muuttujien lkm = yhtälöiden lkm = 3.

$$x = \frac{1}{|A|} \begin{vmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 3 & -1 & -1 \end{vmatrix} = \frac{1}{1} \begin{vmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 3 & -1 & 0 \end{vmatrix} = 1 \cdot (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = -(-1-6) = 7$$

$$y = \frac{1}{|A|} \begin{vmatrix} -11 & 1 & 2 \\ -4 & 2 & 1 \\ 6 & 3 & -1 \end{vmatrix} = \frac{1}{1} \begin{vmatrix} 1 & 7 & 0 \\ 2 & 5 & 0 \\ 6 & 3 & -1 \end{vmatrix} = (-1) \cdot (-1)^{3+3} \begin{vmatrix} 1 & 7 \\ 2 & 5 \end{vmatrix}$$

$$= -(5 - 14) = 9$$

$$z = \frac{1}{|A|}\begin{vmatrix} -11 & 2 & 1 \\ -4 & 0 & 2 \\ 6 & -1 & 3 \end{vmatrix} \, 2 = \frac{1}{1}\begin{vmatrix} 1 & 0 & 7 \\ -4 & 0 & 2 \\ 6 & -1 & 3 \end{vmatrix} = (-1)(-1)^{3+2}\begin{vmatrix} 1 & 7 \\ -4 & 2 \end{vmatrix}$$

$$= 2-(-28) = 30$$

$$\Rightarrow \begin{cases} x = 7 \\ y = 9 \\ z = 30 \end{cases}$$

c)

$$(A|C) = \left(\begin{array}{ccc|c} -11 & 2 & 2 & 1 \\ -4 & 0 & 1 & 2 \\ 6 & -1 & -1 & 3 \end{array}\right) 2 \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ -4 & 0 & 1 & 2 \\ 6 & -1 & -1 & 3 \end{array}\right) 4 \; -6$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 30 \\ 0 & -1 & -1 & -39 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & -1 & -1 & -39 \\ 0 & 0 & 1 & 30 \end{array}\right) 1 \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & -1 & 0 & -9 \\ 0 & 0 & 1 & 30 \end{array}\right) \cdot(-1)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 30 \end{array}\right) \Rightarrow \begin{cases} x = 7 \\ y = 9 \\ z = 30 \end{cases}$$

d) $\begin{cases} -11x + 2y + 2z = 1 \\ -4x + z = 2 \\ 6x - y - z = 3 \end{cases}$

$$\begin{cases} -11x + 2y + 2z = 1 \\ 6x - y - z = 3 \quad |\cdot(2) \end{cases}$$

$$+\begin{cases} -11x + \cancel{2y} + \cancel{2z} = 1 \\ 12x - \cancel{2y} - \cancel{2z} = 6 \end{cases}$$

$$x = 7 \quad \Rightarrow \quad z = 2 + 4x = 2 + 4\cdot 7 = 30$$

$$y = 6x - z - 3 = 6\cdot 7 - 30 - 3 = 42 - 33 = 9$$

$$\Rightarrow \begin{cases} x = 7 \\ y = 9 \\ z = 30 \end{cases}$$

8.

a)
$$\begin{cases} 3x + 4y - 3z = -3 \\ 2x + 3y + 2z = 5 \\ x + y + z = 4 \\ 3x + 4y + 3z = 9 \end{cases}$$

$$(A|C) = \left(\begin{array}{ccc|c} 3 & 4 & -3 & -3 \\ 2 & 3 & 2 & 5 \\ 1 & 1 & 1 & 4 \\ 3 & 4 & 3 & 9 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 2 & 3 & 2 & 5 \\ 3 & 4 & -3 & -3 \\ 3 & 4 & 3 & 9 \end{array}\right) \begin{array}{l} -2 \\ -3 \\ -3 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 1 & -6 & -15 \\ 0 & 1 & 0 & -3 \end{array}\right) \begin{array}{l} -1 \\ -1 \\ -1 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 7 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & -6 & -12 \\ 0 & 0 & 0 & 0 \end{array}\right) \cdot \left(-\frac{1}{6}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 7 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right) -1$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x = 5 \\ y = -3 \\ z = 2 \end{cases}$$

b)
$$\begin{cases} 2x + 3y - 2z = 5 \\ x - 2y + 3z = 2 \\ 4x - y + 4z = 1 \end{cases}$$

$$(A|C) = \left(\begin{array}{ccc|c} 2 & 3 & -2 & 5 \\ 1 & -2 & 3 & 2 \\ 4 & -1 & 4 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 2 & 3 & -2 & 5 \\ 4 & -1 & 4 & 1 \end{array}\right) \begin{array}{l} -2 \\ -4 \end{array} \sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & 7 & -8 & 1 \\ 0 & 7 & -8 & -7 \end{array}\right) -1$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & 7 & -8 & 1 \\ 0 & 0 & 0 & -8 \end{array}\right) \Rightarrow \begin{cases} x - 2y + 3z = 2 \\ 7y - 8z = 1 \\ 0 = -8 \end{cases}$$ id. epätosi

$\Rightarrow$ ei ratkaisua

c) $\begin{cases} x + 2y - z + 3w = 3 \\ 2x + 4y + 4z + 3w = 9 \\ 3x + 6y - z + 8w = 10 \end{cases}$

$$(A|C) = \left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 3 \\ 2 & 4 & 4 & 3 & 9 \\ 3 & 6 & -1 & 8 & 10 \end{array}\right) \begin{array}{c} -2 \\ -3 \end{array} \sim \left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 3 \\ 0 & 0 & 6 & -3 & 3 \\ 0 & 0 & 2 & -1 & 1 \end{array}\right) \begin{array}{c} 3 \\ -3 \end{array}$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 5 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 5 & 0 & 6 \\ 0 & 0 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$\Rightarrow \begin{cases} x + 2y + 5z = 6 \\ 2z - w = 1 \\ 0 = 0 \end{cases} \Rightarrow \begin{cases} z = \frac{1}{5}(6 - x - 2y) \\ w = 2z - 1 \\ x \in \mathbb{R} \\ y \in \mathbb{R} \end{cases}$

d) $\begin{cases} 3x + 4y - 3z = -3 \\ 2x + 3y + 2z = 5 \\ x + y + z = 4 \\ 2x + 2y + 2z = 5 \end{cases}$

$$(A|C) = \left(\begin{array}{ccc|c} 3 & 4 & -3 & -3 \\ 2 & 3 & 2 & 5 \\ 1 & 1 & 1 & 4 \\ 2 & 2 & 2 & 5 \end{array}\right) \begin{array}{c} \\ \\ -2 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 3 & 4 & -3 & -3 \\ 2 & 3 & 2 & 5 \\ 1 & 1 & 1 & 4 \\ 0 & 0 & 0 & -3 \end{array}\right) \Rightarrow 0 = -3 \text{ id. epätosi}$$

$\Rightarrow$ ei ratkaisua.

9

a) $\bar{x}_1 = (1,1,2)$
$\bar{x}_2 = (4,5,5)$
$\bar{x}_3 = (5,8,1)$

Vektorit ovat lineaarisesti riippuvia, jos on olemassa yksi tai useampi $r_i \neq 0$, $i = 1,2,3$ siten, että

$$r_1 \bar{x}_1 + r_2 \bar{x}_2 + r_3 \bar{x}_3 = \bar{0}.$$

Tällöin vektorit voidaan esittää toisten lineaarisena yhdisteenä.

Jos ehdosta $r_1\bar{x}_1 + r_2\bar{x}_2 + r_3\bar{x}_3 = \bar{0}$ seuraa, että $r_1 = r_2 = r_3 = 0$, niin vektorit $\bar{x}_1, \bar{x}_2, \bar{x}_3$ ovat lineaarisesti riippumattomia.

$$r_1(1,1,2) + r_2(4,5,5) + r_3(5,8,1)$$
$$\Leftrightarrow (r_1, r_1, 2r_1) + (4r_2, 5r_2, 5r_2) + (5r_3, 8r_3, r_3) = (0,0,0)$$

$$\Leftrightarrow \begin{cases} r_1 + 4r_2 + 5r_3 = 0 \\ r_1 + 5r_2 + 8r_3 = 0 \\ 2r_1 + 5r_2 + r_3 = 0 \end{cases}$$

Gauss $(A|C) = \left(\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 1 & 5 & 8 & 0 \\ 2 & 5 & 1 & 0 \end{array}\right) \begin{matrix} -1 \\ -2 \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & -3 & -9 & 0 \end{array}\right) \begin{matrix} -4 \\ 3 \end{matrix}$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & -7 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} r_1 - 7r_3 = 0 \\ r_2 + 3r_3 = 0 \end{cases} \Leftrightarrow \begin{cases} r_1 = 7r_3 \\ r_2 = -3r_3 \end{cases}$$

Siis $\begin{cases} r_1 = 7r_3 \\ r_2 = -3r_3 \\ r_3 \in \mathbb{R} \end{cases}$ $\Rightarrow$ Vektorit ovat lineaarisesti riippuvia.

Vast. ei.

b) $\bar{x}_1 = (1,1,2)$
$\bar{x}_2 = (4,5,5)$
$\bar{x}_3 = (-1,-2,2)$

$$r_1(1,1,2) + r_2(4,5,5) + r_3(-1,-2,2) = (0,0,0)$$
$$\Leftrightarrow (r_1, r_1, 2r_1) + (4r_2, 5r_2, 5r_2) + (-r_3, -2r_3, 2r_3) = (0,0,0)$$

$$\Leftrightarrow \begin{cases} r_1 + 4r_2 - r_3 = 0 \\ r_1 + 5r_2 - 2r_3 = 0 \\ 2r_1 + 5r_2 + 2r_3 = 0 \end{cases}$$

Gauss: $(A|C) = \left(\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 1 & 5 & -2 & 0 \\ 2 & 5 & 2 & 0 \end{array}\right) \begin{array}{l} -1 \\ -2 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -3 & 4 & 0 \end{array}\right) \begin{array}{l} -4 \\ 3 \end{array}$

$\sim \left(\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \begin{array}{l} 1 \\ -3 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \Rightarrow \begin{cases} r_1 = 0 \\ r_2 = 0 \\ r_3 = 0 \end{cases}$

$\Rightarrow$ vektorit ovat lineaarisesti riippumattomia

Vast: Kyllä.