Introduction to Probability Theory II

Exercise 2, Autumn 2007

- 1. Let X and Y be independent random variables with means μ_1 and μ_2 , and variances σ_1 and σ_2 , respectively. Present, using these, following
 - a) E(aX + bY), where a and b are constants;
 - b) $D^2(aX + bY)$, where a and b are constants;
 - c) $E\left(\left(\frac{X-Y}{2}\right)^2\right)$.
- 2. Determine the *p*-fractile of random variable X for p = 0.5, p = 0.75 and p = 0.99, when
 - a) $X \sim Tas(0, 1)$,
 - b) $X \sim \text{Exp}(2)$,
 - c) $X \sim N(1/2^{1}/4)$.
- 3. Let P(A) = p. Determine the probability generating function of the indicator 1_A and use this to determine the probability generating function of distribution Bin(n, p).
- 4. Let X be a \mathbb{N} -valued random variable and G the probability generating function of X.
 - a) Calculate G(0) and G(1).
 - b) Express the probability that X is even using G.
- 5. Let X and Y be independent random variables. Determine the conditional distribution

$$P{X = k | X + Y = n}, \text{ for } k = 0, 1, \dots, n,,$$

when

- a) $X \sim Bin(n_1, p)$ and $Y \sim Bin(n_2, p)$,
- b) $X \sim \text{Poisson}(\lambda_1)$ and $Y \sim \text{Poisson}(\lambda_2)$,
- c) $X, Y \sim \text{Geom}(p)$.
- 6. (Jenssen's inequality) Suppose that g is differentiable and its derivative is increasing. Show that, if random variables X and g(X) have expected value, then

$$g(\mathcal{E}(X)) \le \mathcal{E}(g(X)).$$