## Introduction to Probability Theory II

## Exercise 4, Autumn 2007

1. Show that the characteristic function of distribution $\operatorname{Exp}(\lambda)$ is $\phi$, where

$$
\phi(t)=\left(1-\frac{i t}{\lambda}\right)^{-1} \text { for every } t \in \mathbb{R}
$$

2. Determine the characteristic function of random variable $X$, when it's density function is $f$, where

$$
f(x)=1 / 2 e^{-|x|} \text { for every } x \in \mathbb{R} .
$$

3. Let $X$ and $Y$ be independent random variables and $X \sim Y$. Is it possible that

$$
X+Y \sim 2 X ?
$$

4. $\left\{X_{1}, X_{2}, \ldots, X_{10}\right\}$ is a sample from distribution $\operatorname{Tas}(0,1)$. Approximate probability $P\left\{\sum_{k=1}^{10} X_{k}>7\right\}$ using normal approximation.
5. The total price of customers purchases is rounded to nearist 5 cents. The rounding error in single customer's purchases is a random variable whose values are $-2,-1,0,1$ and 2 , each with probability $1 / 5$. Let $X$ be the loss caused by 10000 customers. Calculate probability that $P\{X>2 €\}$ to three decimal places using normal approximation.
6. A book has 500 pages. A typesetting method produces 1000 errors in a book of this size on average.
a) Use Poisson distribution to calculate the probability that single page has less than 2 errors.
b) Let $X$ be the number of pages that have less than 2 errors. Calculate the probability $P\{X>215\}$ using normal approximation.
