## Introduction to Probability Theory II

## Exercise 4, Autumn 2007

1. Show that the characteristic function of distribution  $\text{Exp}(\lambda)$  is  $\phi$ , where

$$\phi(t) = \left(1 - \frac{it}{\lambda}\right)^{-1}$$
 for every  $t \in \mathbb{R}$ .

2. Determine the characteristic function of random variable X, when it's density function is f, where

$$f(x) = \frac{1}{2}e^{-|x|}$$
 for every  $x \in \mathbb{R}$ .

3. Let X and Y be independent random variables and  $X \sim Y$ . Is it possible that

$$X + Y \sim 2X?$$

- 4.  $\{X_1, X_2, \dots, X_{10}\}$  is a sample from distribution Tas(0, 1). Approximate probability  $P\{\sum_{k=1}^{10} X_k > 7\}$  using normal approximation.
- 5. The total price of customers purchases is rounded to nearist 5 cents. The rounding error in single customer's purchases is a random variable whose values are -2, -1, 0, 1 and 2, each with probability  $\frac{1}{5}$ . Let X be the loss caused by 10000 customers. Calculate probability that  $P\{X > 2 \in\}$  to three decimal places using normal approximation.
- 6. A book has 500 pages. A typesetting method produces 1000 errors in a book of this size on average.
  - a) Use Poisson distribution to calculate the probability that single page has less than 2 errors.
  - b) Let X be the number of pages that have less than 2 errors. Calculate the probability  $P\{X > 215\}$  using normal approximation.