Introduction to Probability Theory II

Exercise 5, Autumn 2007

1. Box contains 10 balls. 2 of these are white and 3 are red. Experiment consists of picking 3 balls without replacement. Let X be number of white balls and Y number of red balls in the sample.
   a) Derive the frequency function of the pair (X, Y).
   b) Determine marginal distributions.
   c) Determine conditional distributions.
2. Function f is the density function of a pair of random variables. Determine constant c, when
   a) $f(x) = \begin{cases} cxy, & \text{if } 0 < x < 1, 0 < y < 1, \\ 0 & \text{otherwise;} \end{cases}$
   b) $f(x) = \begin{cases} ce^{-x-y}, & \text{if } 0 < x < y, \\ 0 & \text{otherwise.} \end{cases}$
3. Let the density function f of a pair (X, Y) be as in 2. Are X and Y independent.
4. Let the random variable X have uniform distribution on the interval $]0, 1[$ and let Y be a random variable whose distribution conditional on $X = x$ is uniform on the interval $]0, 1[$
   a) Find the density function of Y and $\mathrm{E}(Y)$.
   b) Find conditional density function $f_X(\cdot|Y = y)$ and conditional expected value $\mathrm{E}(X|Y = y)$.
5. Two points are placed on a line segment randomly and indepently.
   a) Let $0 < x < a$. Calculate the probability that the distance between points is greater than x.
   b) Calculate expected valua of the distance.
6. n points are placed randomly and independently to the unit disk of the plain $\mathbb{R}^2$. Let R be the distance from origin of the point that is nearest to the origin. Determine the density function of the random variable R.