Introduction to Probability Theory I

Exercise 1, Autumn 2008

1* Let Ω be a set and $X \subset \Omega$ and $Y \subset \Omega$. Show that
   a) $(X^{\mathcal{C}})^{\mathcal{C}} = X$;
   b) If $X \subset Y$, then $Y^{\mathcal{C}} \subset X^{\mathcal{C}}$;
   c) $((X \setminus Y) \cup (Y \setminus X))^{\mathcal{C}} = (X \cap Y) \cup (X^{\mathcal{C}} \cap Y^{\mathcal{C}})$;
   d) $X \setminus Y = X \cap Y^{\mathcal{C}}$.

2. Determine $\displaystyle\bigcup_{q\in\mathbb{Q}} \bigcap_{r\in\mathbb{R}_+}]q-r, q+r[$.
3. One of numbers $\{1, 2, \ldots, 1000\}$ is picked randomly. Determine the probability that this number is
   a) divisible by 7;
   b) divisible by 7 and is not divisible by 17.

4* Consider a game there two dice are thrown. Find the propabilities that
   a) sum of the results is 7,
   b) both results are at most 4,
   c) at least one of the results is at most 3.

5. A painted wooden cube is sawn into 1000 small cubes of equal size. Small cubes are mixed and one of them is picked randomly. Find the probability that this cube has exactly two painted sides.
6. Assume that $P(A) = 0.45$ and $P(B) = 0.75$. What can you say of $P(A \cap B)$.

Passing the course

Each exercise contains two questions marked with an asterisk (*). You have to solve at least four of these questions from Exercises 1–4 and four from Exercises 5–8 to be able to pass the course with two exams. You get one point for each solved question above this minimum. The grade is determined by the sum of exam points and exercise points.