Introduction to Probability Theory I

Exercise 2, Autumn 2008

1* Show that
$$P(A) \leq 1 - P(A^{\mathcal{C}} \cap B^{\mathcal{C}}) \leq P(A) + P(B)$$
and
$$1 - P(A^{\mathcal{C}}) - P(B^{\mathcal{C}}) \leq P(A \cap B) \leq P(A)$$
for any events A and B.

2. Three stochasticians have agreed to meet in the Grand Hotel, but the town has four Grand Hotels. Find the probability that
a) everyone is in the same hotel,
b) all are in different hotels.

3. a) Calculate how many ways there are for 15 persons to sit around a round table. Two seatings are considered to be same if one is a rotation of other.
b) Assume that these 15 persons represent 15 members of UN Security Council. Find the probability that representaties of England and France are seated adjacent to each other and representatives of USA and China are not adjacent.

4* k members of a committee are selected from n candidates. One of of k members is elected as the chairman. How many ways there are to form the committee, if we consider committees with same members but different chairmen to be different? Find the number by
a) first selecting committee members and then the chairman.
b) first selecting the chairman and then selecting the rest of committee.
Check that you obtain same result in both cases!

5. A teacher lectures a course three times every year for 40 years. He tells 3 jokes every time. How many jokes he has to know, if he does not want to tell exactly same three jokes twice? How many jokes he has to know if he tells four jokes on each course?