Introduction to Probability Theory I

Exercise 4, Autumn 2008

1* We perform an experiment, where we throw an weighted tetrahedron whose sides have been numberd from 1 to 4. We observe which side falls against the table. Ratio of frequences that each side appears is seen to be $2:3:4:5$. Construct a probability space that describes this experiment and find probabilities $p_k = P(\{\omega\})$ for each point $\omega \in \Omega$.

2. Let $\Omega = \{1,2,3,4,5,6,7,8,9,10\}$. Define probabilities $p_k = P(\{k\})$ for each point $\omega \in \Omega$ such that
   a) p_k is proportional to k,
   b) p_k is proportional to $\ln k$.
   Let $A = \{k \in \Omega \mid k > 6\}$ and $B = \{k \in \Omega \mid k = z^2 \text{ for some } z \in \mathbb{Z}\}$. Find the probabilities of A and B in each case.

3. Population consists of 818 people. 276 have been vaccinated against an epidemic. 69 people fall ill. Three of those, who were vaccinated.
   a) Find the conditional probability of a person falling ill given that he is vaccinated.
   b) Find the conditional probability of a person being vaccinated given that he did not fall ill.

4* 5 cards are drawn from a shuffled deck of cards. Find the conditional probability of getting at least one ace given that all cards are at least 10. (Ace is 14.)

5. Show that, if $P(A) = P(B) = \frac{3}{4}$, then $P(A|B) \geq \frac{2}{3}$.

6. 70% of email Johanna gets is spam. A spam filter classifies 75% of spam correctly. Find the probability that an arriwing email is correctly classified as spam.