Matematiikan perusmetodit/mat.

Harjoitus 14 syksy 2009

A osa:

1. Integroi

   a) $\int \cos 6x\, dx$, b) $\int \sin(4x+1)\, dx$, c) $\int \sin^5 x \cos x\, dx$, d) $\int \tan^2 x\, dx$.

2. Integroi

   a) $\int \sin^2 x\, dx$, b) $\int \cos^2 x\, dx$, c) $\int \cos^3 x\, dx$, d) $\int \sin^5 x\, dx$, e) $\int \cos^4 x\, dx$.

3. Integroi

   a) $\int \frac{1}{1+a^2x^2}\, dx$, b) $\int \frac{dx}{2+x^2}$, c) $\int \frac{x^2}{x^2+1}\, dx$, d) $\int \frac{x+1}{x^2-x+1}\, dx$.

4. Integroi

   a) $\int x \sin x\, dx$, b) $\int x^2 \sin x\, dx$, c) $\int (2x+1) \sin 2x\, dx$, d) $\int x \sin x \cos^3 x\, dx$.

5. Integroi

   a) $\int xe^{2x}\, dx$, b) $\int xe^{-\frac{x}{2}}\, dx$, c) $\int x^2 e^{2x}\, dx$, d) $\int x^3 e^x\, dx$.

6. Integroi

   a) $\int x \ln x\, dx$, b) $\int \ln x\, dx$, c) $\int \ln x^2\, dx$, d) $\int x^2 \ln x\, dx$, e) $\int \frac{\ln(1+x^2)}{x^2}\, dx$, f) $\int \ln(1+x^2)\, dx$.

7. Integroi

   a) $\int \arctan\, dx$, b) $\int \arcsin x\, dx$, c) $\int \cos(\ln x)\, dx$, d) $\int e^x \cos x\, dx$.

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B osa:

1. Integroi suluissa annettua sijoitusta käyttäen

   a) $\int \frac{1}{\sqrt{9-x^2}}\,dx$, $(x = 3\sin t,\ -\frac{\pi}{2} < t < \frac{\pi}{2})$,

   b) $\int \frac{1}{1+\sqrt{x+1}}\,dx$, $(t = \sqrt{x+1})$,

   c) $\int \frac{1}{1+\sqrt[3]{x+1}}\,dx$, $(t = \sqrt[3]{x+1})$,

   d) $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}\,dx$, $(x = t^6,\ t > 0)$.

2. Integroi

   a) $\int \frac{dx}{x(x-1)}$, b) $\int \frac{1+x^2}{x(1+x)}\,dx$, c) $\int \frac{2x^2+8x-2}{(x-1)^2(x+1)^2}\,dx$.

3. Integroi

   a) $\int \frac{1}{1+\sqrt{x-1}}\,dx$, b) $\int \sqrt{2-x^2}\,dx$, c) $\int \frac{x}{(3x-1)\sqrt{3x-1}}\,dx$, d) $\int \frac{x}{x+\sqrt{x}}\,dx$,
   e) $\int \frac{4^x+1}{2^x+1}\,dx$, f) $\int \sqrt{x^2+2}\,dx$.

4. Johda osittaisintegroinnin avulla palautuskaavat

   a) $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$, $n = 2, 3, \ldots$,

   b) $\int \cos^n x\,dx = -\frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$, $n = 2, 3, \ldots$.