Introduction to Probability Theory I

Exercise 3, Autumn 2009

1. An urn contains 15 balls. Five of those are white and the rest black. We pick 10 balls without replacement. Find the probability that we pick
   a) at least one white ball;
   b) all five of the white balls.
2. Set E has N elements. We pick a sample of n elements. Find the probability that an element a is in the sample when sample is chosen
   a) without replacement,
   b) with replacement.
3. A 1500 meter race has 36 competitors. They are randomly divided to three heats, 12 runners in each heat. Find the probability that Finnish Aalto and Kenyan Korir are in same heat.
4. Group consists of $2n$ boys and $2n$ girls. It is divided into to groups of $2n$ children. Find the probability that both groups have equally many boys and girl and approximate this probability using Stirling's formula.
5. We place N balls, that nunbered from 1 to N, in an urn. We pick n ball from this urn. Find the probability that the largest picked number is k, when we pick the sample
   a) with replacement,
   b) without replacement.
   (Hint: consider events B_k="largest number is less or equal to k").
6. Assume that an urn contains N balls, and K balls of these are white. Assume that k and n are integers such that $k \leq K$ and $n \leq N$. Let P_1 be the probability when sample is chosen without replacement and P_2 when sample is chosen with eplacement. Let

   $$A_k = \text{"Sample contains exactly } k \text{ with balls"}.$$

   Show that

   $$\frac{P_1(A_k)}{P_2(A_k)} \to 1,$$

   if $K \to \infty$, $N \to \infty$ and $N - K \to \infty$.