Introduction to Probability Theory I

Exercise 4, Autumn 2009

1. 5 cards are drawn from a shuffled deck of cards. Find the conditional probability of getting at least one ace given that all cards are at least 10. (Ace is 14.)
2. Population consists of 818 people. 276 have been vaccinated against an epidemic. 69 people fall ill. Three of those were vaccinated.
   a) Find the probability of a person falling ill given that he is vaccinated.
   b) Find the probability of a person being vaccinated given that he did not fall ill.
3. Show that, if $P(A) = P(B) = \frac{2}{3}$, then $P(A|B) \geq \frac{2}{3}$.
4. Consider an experiment that consists of throwing two dice. Let

$$A = \text{"sum of results is 6"},$$
$$B = \text{"result of the first die is 4"}.$$

   Are events A and B independent? What happens if A is replaced by event "sum of results is 7"?
5. Let A and B be events and $P(A) \neq 0$. Show that, if $P(B|A) = P(B|A^C)$, then the events A and B are independent.
6. A factory produces a device that can have three different faults: A, B and C. Occurence of different faults is independent of each other. Find the probability that
   a) all three faults occur,
   b) no faults occure,
   c) fault A does not occur but fault B or C does occur,
   d) at most one fault occurs.
7. Assume that A, B ja C are events and $P(A) = P(B) = P(C) = 0,25$. Find $P(A \cup B \cup C)$, if
   a) A, B and C are independent;
   b) A and B are independent, A and C are independent, and B are C mutually exclusive events.
8. Give an example of three events A, B and C that are not independent, even tought

$$P(A \cap B \cap C) = P(A)P(B)P(C).$$