Introduction to Probability Theory I

Exercise 5, Autumn 2009

1. Mr K claims that he can use a magic wand to find water. The neighbours decide to test his claim. Ten times they give him two barrels to test, one empty and other filled with water. Mr K must detect the barrel filled with water. Find the probability that Mr K picks correct barrel at least eight times, if he
   a) picks barrel randomly,
   b) picks correct barrel with probability 0.8.
2. N children toss a coin to determine one seeker, while others hide. The seeker is only child to have different result from all other children. Find the probability that seeker is solved on round n.
3. (*Banach's match problem*) Suppose a mathematician carries two matchboxes at all times: one in his left pocket and one in his right. Each time he needs a match, he is equally likely to take it from either pocket. Suppose he reaches into his pocket and discovers that the box picked is empty. If it is assumed that each of the matchboxes originally contained n matches, what is the probability that there are exactly r matches in the other box? Find the probability if $n = 50$ and $r = 0$.
4. A forgetfull gentleman forgets his umbrella to a store with probability $^1/_4$. One day he visits four stores and notices what he has forgotten his umbrella to one of those. Find the probability that the umbrella is in each of these four stores.
5. A lost letter has probability $^1/_2$ of being in one of six drawers. Find the probability that letter is in the sixth drawer given that first five drawers have been searched in vain.
6. Three chests contain following coins:
   1: two gold coins,
   2: one gold and one silver coin,
   3: two silver coins.
   First we choose a chest randomly, then we pick a coin from this chest randomly. The coin tunrs out to be a gold coin. What is the conditional probability that other coin in the chest is also golden?
7. Consider three words: Schwarzwald, employee and mayor. We pick one of these randomly. Then we pick a letter from this word. Find the probability

that the word is English, given that the letter is a vovel.