

Introduction to Probability Theory I

Exercise 6, Autumn 2009

1. Find the distribution of random variable X if
 - a) X is the number of faulty products in a box of 48 products. Assume that each item has probability 0.05 of being faulty independent of other items in the box;
 - b) X is the number of aces when drawing 13 cards from a deck without replacement;
 - c) X is the number of balls in a certain box when k balls are distributed randomly into n boxes;
 - d) X is the number of unsuccessful trials before a pair of sixes appear when throwing a pair of dice.
 - e) X is the number of colourblind persons in a 10 person sample from a population of 100 people. Assume that sample is chosen with replacement.
2. Find the probability that value of random variable X is even, if the distribution of X is
 - a) geometric with parameter p ,
 - b) binomial with parameters n and p , and
 - c) Poisson distribution with parameter λ .
3. Consider a game, where a coin is tossed repeatedly until both heads and tails have appeared at least twice. Let X be the number of round where the game ends. Find the distribution and frequency function of X . Furthermore, find the smallest value of $n \in \mathbb{N}$ such that $P(\{X \leq n\}) > 0.9$.
4. A message consists of 100 bits (each has value 0 or 1). In every transmission each bit is changed with probability $p = 0.001$ independent of other bits. Find the probability that the message is in it's original form after ten subsequent transmissions.
5. Let $r > 0$. A dartboard consists of 10 concentric rings those radii are $r, 2r, 3r, \dots, 10r$. Outmost ring is worth one point, next ring is worth two points and so on. The innermost ring is worth ten points. Assume that the probability of dart hitting any part of the board is proportional to the area of that part. Find the expectation of the points awarded for a single dart.