Introduction to Probability Theory I

Exercise 7, Autumn 2009

1. Let X be a random variable whose probability density function is

$$f(x) = \begin{cases} cxe^{-x}, & \text{if } x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

   a) Determine constant c.
   b) Find the distribution function.
   c) Determine $P\{0 < X < 1\}$.

2. Consider an undergroud train that is scheduled to depart at 3, 5, 8, 10, 13, 15, 18, 20, 23, 25, ... minutes past 8. Persons arrival time on station is distributed uniformly from 7:02 to 7:24. Find the probability that she has to wait at most one minute.
3. Time, in minutes, a person has to wait for a tram is distributed uniformly on interval $]0, 10[$. Find the probability that a person, who has already waited for 4 minutes, has to wait still at least x minutes.
4. A factory produces a device those useable lifetime in years follows exponential distribution with parameter λ, where $\lambda > 0$. Find λ such that the probability for lifetime being less than 3 years is more than 0,5.
5. Time, in minutes, a customer uses in a bank follows exponential distribution with parameteer $\lambda = {}^1\!/_{10}$.
   a) Find the probability that customer stays in the bank more than 15 minutes.
   b) Find the probability that customer, who has already been in the bank for 10 minutes, stays in more than 15 minutes.
6. Weight of envelopes in grams is distributed normally with expectation 1.95 and variance 0.05.
   a) Find the probability that weight of a randomly selected envelope is more than 1.8 grams and less than 2.1 grams.
   b) Find the probability that weight of a randomly selected envelope is more than 2.1 grams.
   c) Find the expected value of number of envelopes weighting more 2.1 grams in a pack of 100 envelopes.
7. Find the expectation $\mathrm{E}(X)$, if random variable X is continuously distributed with distribution function f, and
   a) $f(x) = {}^1\!/_2 e^{-|x|}$, for every $x \in \mathbb{R}$;
   b) $f(x) = {}^8\!/_{x^3}$, for every $x > 2$;
   c) $f(x) = xe^{-{}^1\!/_2 x^2}$, for every $x > 0$.