Introduction to Probability Theory I

Exercise 8, Autumn 2009

1. Find the expectation $\mathrm{E}(X)$, if the probability density function of random variable X is f, and

   a) $f(x) = \begin{cases} \frac{8}{x^3}, & x > 2, \\ 0 & \text{otherwise;} \end{cases}$

   b) $f(x) = \frac{1}{2}e^{-|x|}$, $x \in \mathbb{R}$;

   c) $f(x) = \begin{cases} x\mathrm{e}^{-\frac{1}{2}x^2}, & x > 0, \\ 0 & \text{otherwise.} \end{cases}$

2. Asssume that X_1, X_2 and X_3 are independent normally distributed random variables and that each has distribution $\mathrm{N}(1, 3)$. Find

$$P\{X_1 + X_2 + X_3 > 0\}.$$

3. Let X_1, X_2, $\ldots$, X_n be measuring errors in repeated measurements. Assume that random variables X_1, X_2, $\ldots$, X_n are independent, normally distributed, each has distribution $\mathrm{N}(0, \sigma^2)$ and

$$P(|X_i| < a) = 0,95 \text{ for every } i = 1, 2, \ldots, n.$$

   Let $\bar{X}$ be the average of X_i ie $\bar{X} = {}^1\!/_n \sum_{i=1}^{n} X_i$. Find n such that

$$P\{|\bar{X}| < \frac{a}{100}\} = 0,95?$$

4. Determine the distribution of random variable $X - Y$, if X and Y are independent and both follow exponential distribution with parameter λ, where $\lambda > 0$.
5. Find the distribution of random variable $2X^2 + 1$, if $X \sim \mathrm{N}(0, 1)$.
6. A ray of light originating from point $(0, 1) \in \mathbb{R}^2$ forms angle Θ with x-axel. Assume that Θ is uniformly distributed on interval $]-\frac{\pi}{2}, \frac{\pi}{2}[$. Let X be the x coordinate of the intersection of light ray and x-axel. Find the probability function and probability density function of X. Does X have an expectation?