# Introduction to Probability Theory II

## Exercise 1, Autumn 2009

1. Of 15 lottery tickets
   - three win 10€ and
   - four win 3€.

   Rest of the tickets are empty. Let $X$ be the "amount you win with two tickets". Find the expectation $E(X)$.
2. Consider $n$ real numbers. The numbers are rounded to nearest integer. The errors in rounding are independent random variables that are uniformly distributed on interval $]-1/2, 1/2]$. Let $X$ be the error in arithmetic mean calculated using rounded numbers in stead of original. Find $n$ such that
   $$P\{|X| \geq 0,01\} < 0,05.$$
   Use Chebyshev inequality to approximate the error.
3. Show that events $A_1$, $A_2$, ..., $A_n$ are independent if and only if their indicators are independent random variables.
4. Show that events $A$ and $B$ are independent if and only if for $\mathrm{Cov}(\mathbf{1}_A, \mathbf{1}_B) = 0$, where $\mathbf{1}_A$ and $\mathbf{1}_B$ are the indicators of events $A$ and $B$ respectively.
5. Let $X$ be a random variable that is uniformly distributed on interval $]0, 1[$. Find the expectations $\mathrm{E}(X^2)$ and $\mathrm{E}(\sin(2\pi X))$.
6. Let $X$ and $Y$ be normally distributed random variables, whose distribution is $\mathrm{N}(0, 1)$. Consider a triangle whose vertices are the origin and points $(X, 0)$ and $(0, Y)$. Find the expecctation of the area of this triangle.