Introduction to Probability Theory II

Exercise 3, Autumn 2009

1. There has been supernova explosions in our galaxy in 1987, 1604, 1572 and 1054. It is believed that, on average, explosions happen once in 300 years. Suppose that the explosions form a Poisson process. Determine the probability that
   a) in a certain 60 year interval there are at least two supernova explosions,
   b) in a certain 450 year interval there are no supernova explosions.
2. The arrival of long distance phone calls forms a Poisson process. There has been one call before moment $t > 0$. Let T be the arrival time of this call. Determine the distribution of random variable T.
   Hint: Calculate $P\{T \leq s \mid X([0,t]) = 1\}$ *for* $0 < s < t$.
3. Mr K has a shop in connection with his apartement. He has on average 6 customer per hour. The arrival of a customer is announced by a bell. Mr K has decided to serve his customers after every nth bell sounding. What is the probability that Mr K can do a chore, that requires 10 minutes, without interruptions during his break?
4. Let the density function of random variable X be f, where
   a) $f(x) = \begin{cases} \frac{c}{\sqrt{x}} e^{-\frac{x}{2}} \text{ if } x > 0, \\ 0 \text{ otherwise;} \end{cases}$
   b) $f(x) = \begin{cases} c x^5 e^{-2x} \text{ if } x > 0, \\ 0 \text{ otherwise.} \end{cases}$
   Determine constant c, the name of distribution, expected value $\mathrm{E}(X)$ ja variance $\mathrm{D}^2(X)$.
5. Determine $\mathrm{E}(X^{-k})$ when $X \sim \mathrm{Gamma}(r, \lambda)$ and $k \in \mathbb{N}_+$. When X^{-k} has an expected value?
6. The velocities of molecule in the direction of each of $x-$, $y-$ and $z-$axis are independent random variables whose distribution is $\mathrm{N}(0, \sigma^2)$. Determine the density function of molecules speed. For your information: $\Gamma(^1/_2) = \sqrt{\pi}$. *Maxwell derived this distribution starting from assumption that the distribution has to be invariant under all rotations of a 3–dimensional space.*