

## Introduction to Probability Theory II

### Exercise 4, Autumn 2009

1. Show that the characteristic function of distribution  $\text{Exp}(\lambda)$  is  $\phi$ , where

$$\phi(t) = \left(1 - \frac{it}{\lambda}\right)^{-1} \text{ for every } t \in \mathbb{R}.$$

2. Determine the characteristic function of random variable  $X$ , when it's density function is  $f$ , where

$$f(x) = \frac{1}{2}e^{-|x|} \text{ for every } x \in \mathbb{R}.$$

3.  $\{X_1, X_2, \dots, X_{10}\}$  is a sample from distribution  $\text{Uas}(0, 1)$ . Approximate probability  $P\{\sum_{k=1}^{10} X_k > 7\}$  using normal approximation.
4. The total price of customers purchases is rounded to nearest 5 cents. The rounding error in single customer's purchases is a random variable whose values are  $-2, -1, 0, 1$  and  $2$ , each with probability  $\frac{1}{5}$ . Let  $X$  be the loss caused by 10 000 customers. Calculate probability that  $P\{X > 2\text{€}\}$  to three decimal places using normal approximation.
5. A factory produces two kinds of machine parts, hollow and filled cylinders. Filled cylinders are intended to fit inside hollow ones. Let  $X$  be the outer diameter of filled cylinder and  $Y$  inner diameter of hollow cylinder. Assume that  $X \sim N(10.40, 0.03^2)$  and  $Y \sim N(10.52, 0.04)$ .
  - a) Pick one filled and one hollow cylinder randomly. Find the probability that filled cylinder fits inside hollow one.
  - b) Pick a hollow and filled cylinder hundred times. Let  $N$  be the number of these pairs, where filled cylinder does not fit inside the hollow one. Use normal approximation to approximate the probability  $P\{N \leq 1\}$ .
  - c) Use Poisson distribution to approximate the probability  $P\{N \leq 1\}$ .
  - d) Compare values from b) and c) to the correct value of  $P\{N \leq 1\}$ .
6. A book has 500 pages. A typesetting method produces 1000 errors in a book of this size on average.
  - a) Use Poisson distribution to calculate the probability that single page has less than 2 errors.
  - b) Let  $X$  be the number of pages that have less than 2 errors. Calculate the probability  $P\{X > 215\}$  using normal approximation.