

Introduction to Probability Theory II

Exercise 5, Autumn 2009

- Box contains 10 balls. 2 of these are white and 3 are red. Experiment consists of picking 3 balls without replacement. Let X be number of white balls and Y number of red balls in the sample.
 - Derive the frequency function of the pair (X, Y) .
 - Determine marginal distributions.
 - Determine conditional distributions.
- Function f is the density function of a pair of random variables. Determine constant c , when
 - $f(x, y) = \begin{cases} cxy, & \text{if } 0 < x < 1, 0 < y < 1, \\ 0 & \text{otherwise;} \end{cases}$
 - $f(x, y) = \begin{cases} ce^{-x-y}, & \text{if } 0 < x < y, \\ 0 & \text{otherwise.} \end{cases}$
- Let the density function f of a pair (X, Y) be as in 2 a). Are X and Y independent.
- Let the density function f of a pair (X, Y) be as in 2 a). Find conditional density functions $f_X(\cdot | Y = y)$ and $f_Y(\cdot | X = x)$.
- Let the random variable X have uniform distribution on the interval $]0, 1[$ and let Y be a random variable whose distribution conditional on $X = x$ is uniform on the interval $]0, 1[$
 - Find the density function of Y and $E(Y)$.
 - Find conditional density function $f_X(\cdot | Y = y)$ and conditional expected value $E(X | Y = y)$.
- n points are placed randomly and independently to the unit disk of the plain \mathbb{R}^2 . Let R be the distance from origin of the point that is nearest to the origin. Determine the density function of the random variable R .