

Matematiikan perusteet taloustieteilijöille I

Harjoitus 1, syksy 2013

1. a) $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

b) $\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$

c) $\frac{1}{2} : \frac{1}{4} = 2$

d) $2^3 - (-2)^3 = 8 - (-8) = 16$

e) $(-3)^4 + \sqrt{36} - \sqrt[3]{-27} = 81 + 6 - (-3) = 90$

f) $27^{-\frac{1}{3}} + \sqrt[3]{-27} = \frac{1}{3} + 3 = \frac{10}{3}$

g) $\sqrt{-36}$ ei ratkaisua, koska $-36 < 0$

2. a) $(x + 3)^2 = x^2 + 6x + 9$

[muistisääntö: $(a + b)^2 = a^2 + 2ab + b^2$]

tai $(x + 3)^2 = (x + 3)(x + 3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9$

b) $(x - 3)^2 = (x + (-3))^2 = x^2 - 6x + 9$

[muistisääntö: $(a + b)^2 = a^2 + 2ab + b^2$]

tai $(x - 3)^2 = (x - 3)(x - 3) = x^2 - 3x - 3x + 9 = x^2 - 6x + 9$

c) $(-x - 3)^2 = ((-x) + (-3))^2 = x^2 + 6x + 9$

[muistisääntö: $(a + b)^2 = a^2 + 2ab + b^2$]

tai $(-x - 3)^2 = (-x - 3)(-x - 3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9$

d) $(x - 3)(x + 3) = x^2 - 9$

[muistisääntö: $(a - b)(a + b) = a^2 - b^2$]

tai $(x - 3)(x + 3) = x^2 + 3x - 3x - 9 = x^2 - 9$

e)

$$\begin{aligned} \frac{1}{x+2} + \frac{2x}{x-1} &= \overset{x-1}{x+2} \frac{1}{x+2} + \overset{x+2}{x-1} \frac{2x}{x-1} \\ &= \frac{x-1}{(x-1)(x+2)} + \frac{2x^2+4x}{(x-1)(x+2)} \\ &= \frac{x-1+2x^2+4x}{x^2+2x-x-2} \\ &= \frac{2x^2+5x-1}{x^2+x-2} \end{aligned}$$

f) $\frac{1}{x+2} \cdot \frac{2x}{x-1} = \frac{2x}{(x+2)(x-1)} = \frac{2x}{x^2+x-2}$

g) $\frac{1}{x+2} : \frac{2x}{x^2-4} = \frac{1}{x+2} \cdot \frac{x^2-4}{2x} = \frac{1}{x+2} \cdot \frac{(x+2)(x-2)}{2x} = \frac{x-2}{2x}$

h) $x^2 \cdot \sqrt[4]{\left(x^{-\frac{8}{3}}\right)^3} y^8 = x^2 \cdot \sqrt[4]{x^{-8} \cdot y^8} = \sqrt[4]{(x^2)^4 \cdot x^{-8} y^8} = \sqrt[4]{x^8 x^{-8} y^8}$
 $= \sqrt[4]{x^0 \cdot y^8} = \sqrt[4]{y^8} = \sqrt[4]{y^4 \cdot y^4} = y \cdot y = y^2$

3.

$$\begin{aligned}(\sqrt{x} + \sqrt{y})(\sqrt[4]{x} + \sqrt[4]{y})(\sqrt[4]{x} - \sqrt[4]{y}) &= (\sqrt{x} + \sqrt{y}) \left((\sqrt[4]{x})^2 - (\sqrt[4]{y})^2 \right) \\ &= (\sqrt{x} + \sqrt{y}) \left((x^{\frac{1}{4}})^2 - (y^{\frac{1}{4}})^2 \right) \\ &= (\sqrt{x} + \sqrt{y})(x^{\frac{1}{2}} - y^{\frac{1}{2}}) \\ &= (\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) \\ &= (\sqrt{x})^2 - (\sqrt{y})^2 \\ &= x - y\end{aligned}$$

4. Tiedetään kaksi pistettä, jotka ovat suoralla: origo $(0, 0)$ ja $(\frac{13}{9}, \frac{17}{6})$.

Valitaan: $x_1 = 0$, $y_1 = 0$, $x_2 = \frac{13}{9}$ ja $y_2 = \frac{17}{6}$.

Kulmakerroin

$$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{17}{6} - 0}{\frac{13}{9} - 0} = \frac{17}{6} \cdot \frac{9}{13} = \frac{51}{26}$$

Suoran yhtälö

$$\begin{aligned}y - y_1 &= a(x - x_1) \\ y - 0 &= \frac{51}{26} \cdot (x - 0) \\ y &= \frac{51}{26}x\end{aligned}$$

Siis suoran yhtälö on $y = \frac{51}{26}x$.

5. Ratkaise yhtälöt

a)

$$\begin{aligned}x + 2 &= 0 && | - 2 \\x &= -2\end{aligned}$$

b)

$$\begin{aligned}-2x + 10 &= 0 && | - 10 \\-2x &= -10 && | : -2 \\x &= 5\end{aligned}$$

6. Ratkaise epäyhtälöt

a)

$$\begin{aligned}x + 2 &> 0 && | - 2 \\x &> -2\end{aligned}$$

b)

$$\begin{aligned}-2x + 10 &< 0 && | - 10 \\-2x &< -10 && | : -2 \\x &> 5\end{aligned}$$