# Matematiikan perusteet taloustieteilijöille Ib

Harjoitus 5, syksy 2013

1. a)

$$f(x,y) = 2x^5y - xy^3$$

$$f_x = 10x^4y - y^3$$

$$f_y = 2x^5 - 3xy^2$$

b)

$$f(x,y) = x^y + y^x$$

$$f_x = yx^{y-1} + y^x \ln y$$

$$f_y = x^y \ln x + xy^{x-1}$$

c)

$$f(x,y) = \ln(x^2+y^2) = \ln(x^2+y^2)$$

$$f_x = \cdot\frac{1}{x^2+y^2} \cdot D_x(x^2+y^2) = \frac{1}{x^2+y^2} \cdot 2x = \frac{2x}{x^2+y^2}$$

$$f_y = \cdot\frac{1}{x^2+y^2} \cdot D_y(x^2+y^2) = \frac{1}{x^2+y^2} \cdot 2y = \frac{2y}{x^2+y^2}$$

d)

$$f(x,y,z) = (2x^2 + y^3)^2 + e^{2z}$$

$$f_x = 2(2x^2 + y^3) \cdot 4x = 8x(2x^2 + y^3)$$

$$f_y = 2(2x^2 + y^3) \cdot 3y^2 = 6y^2(2x^2 + y^3)$$

$$f_z = e^{2z} \cdot 2 = 2e^{2z}$$

2.

$$f(x,y) = \begin{cases} -x + y^2, & x < 0 \\ x + y^2, & x \geq 0 \end{cases}$$

Kun $x < 0$, niin $f(x,y) = -x + y^2$ on polynomina jatkuva muuttujien $x$ ja $y$ suhteen.

Kun $x > 0$, niin $f(x,y) = x + y^2$ on polynomina jatkuva muuttujien $x$ ja $y$ suhteen.

Tarkastellaan funktion jatkuvuutta kohdassa $x = 0$:

$$\lim_{x \to 0^-} f(x,y) = \lim_{x \to 0^-} (-x + y^2) = -0 + y^2 = y^2$$
$$\lim_{x \to 0^+} f(x,y) = \lim_{x \to 0^+} (x + y^2) = 0 + y^2 = y^2 = f(0,y)$$

Raja-arvo $\lim_{x \to 0} f(x,y)$ on olemassa, kun $y^2 = y^2$, joka on aina tosi. Siis funktio $f(x,y)$ on jatkuva.

3.

$$f(x,y) = x^2y^5 \qquad\qquad (x,y) = (x_0 + \Delta x, y_0 + \Delta y)$$

alkutilanne: $(x_0, y_0) = (1, 2)$ $\qquad = (1 + 0,5\ ;\ 2 - 0,2)$

muutos: $\Delta x = 0,5$ ja $\Delta y = -0,2$ $\qquad = (1,5\ ;\ 1,8)$

Todellinen muutos:

$$\begin{aligned}\Delta f &= f(x,y) - f(x_0,y_0) = f(1,5\ ;\ 1,8) - f(1,2) \\ &= (1,5)^2(1,8)^5 - 1^2 \cdot 2^5 = 10,51528 \approx 10,5\end{aligned}$$

Differentiaali:

$$\frac{\partial f}{\partial x} = 2xy^5 \qquad\qquad \frac{\partial f}{\partial y} = 5x^2y^4$$

$$\begin{aligned}df(1,2) &= \frac{\partial f}{\partial x}(1,2)\Delta x + \frac{\partial f}{\partial y}(1,2)\Delta y \\ &= 2 \cdot 1 \cdot 2^5 \cdot 0,5 + 5 \cdot 1^2 \cdot 2^4 \cdot (-0,2) = 16\end{aligned}$$

4. $f(x,y) = x^2 - 3y$, missä $x = uv$ ja $y = u^2 + v^2$.

$$\begin{aligned}f(u,v) &= (uv)^2 - 3(u^2 + v^2) \\ &= u^2v^2 - 3u^2 - 3v^2\end{aligned}$$

$$\frac{\partial f}{\partial u} = 2uv^2 - 6u$$
$$\frac{\partial f}{\partial v} = 2u^2v - 6v$$

5. $f(x,y,z) = x^3y^2 + e^{3y} + z^2$

Laskettavat derivaatat: $f_{xx}$, $f_{yy}$, $f_{zz}$, $f_{xy}$, $f_{yx}$, $f_{xz}$, $f_{zx}$, $f_{yz}$, $f_{zy}$

$$f_x = 3x^2y^2$$
$$f_y = 2x^3y + e^{3y} \cdot 3 = 2x^3y + 3e^{3y}$$
$$f_z = 2z$$

$$f_{xx} = 6xy^2$$
$$f_{yy} = 2x^3 + 3e^{3y} \cdot 3 = 2x^3 + 9e^{3y}$$
$$f_{zz} = 2$$

$$f_{xy} = 6x^2y$$
$$f_{yx} = 6x^2y$$

$$f_{xz} = 0$$
$$f_{zx} = 0$$

$$f_{yz} = 0$$
$$f_{zy} = 0$$