DISCRETE MATHEMATICS

Test 1, 7.3.2007

1. a) Let A, B and C be sets. Prove that $(A \cap B) \cup C = A \cap (B \cup C)$ if and only if $C \subseteq A$.

   b) Determine $R \circ S$, when R and S are the following relations:

$$R = \{(1, a), (1, b), (2, a), (3, b)\} \subseteq \{1, 2, 3\} \times \{a, b\},$$
$$S = \{(a, 1), (a, 3), (b, 3)\} \subseteq \{a, b\} \times \{1, 2, 3\}.$$

2. a) Let R be a relation on a set X. Prove that $(R^n)^{-1} = (R^{-1})^n$ for all $n \in \mathbb{Z}_+$.

   b) Prove that if we choose 14 different numbers from the set $\{1, 2, \ldots, 25\}$ then among them there are two whose sum equals 26.

3. A party has four kinds of beer: Heineken, Guiness, Fosters and Budweiser, at least 12 bottles each. In how many ways they can choose to drink from these

   a) ten bottles with no limitation;

   b) twelve bottles such that there is at least one Heineken, an even number of Guiness and at most five bottles of Budweiser?

4. Let $S = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (1, 3), (2, 3), (3, 2), (3, 3), (3, 4)\} \subseteq S \times S$. Five new pairs from $S \times S$ are added to R at random. What is the propability that this new R

   a) is a relation on S;

   b) is a reflective relation on S;

   c) contains the transitive closure $t(R)$ as its subset, when we know that at least one required pair was selected?