TEST-1: "Introduction to partial differential equations", 20.10.2008

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1) Assume that $x_1 > 0$. Solve by the method of characteristics the initial value problem and determine the values x_1 and x_2 for which it exists:

$$x_2 \partial_1 u(x_1, x_2) + x_1 \partial_2 u(x_1, x_2) = u(x_1, x_2), \quad u(x_1, 1) = 1.$$

2) Solve by the method of separation of variables the heat conductor problem in a bar of lenth 10 cm for which diffusivity is 1. Suppose that the ends of the bar are insulated and the initial temperature distribution of the bar is $f(x) = x^2$. Prove that the temperature distribution is C^∞-function for $t > 0$. Find the steady state temperature distribution of the bar.

3) Solve by the method of separation of variables the following boundary value problem with Cauchy data for one-dimensional wave equation:

$$u_{tt}(x,t) = u_{xx}(x,t), \quad 0 < x < 1, \quad t > 0$$

$$u(0,t) = 0, \quad u_x(1,t) = 0, \quad t \geq 0$$

$$u(x,0) = 0, \quad u_t(x,0) = \sin\frac{\pi x}{2}, \quad 0 \leq x \leq 1.$$

4) Solve by the method of separation of variables the following boundary value problem for two-dimensional Laplace equation on the rectangle:

$$\Delta u(x,y) = 0, \quad 0 < x < 1, \quad 0 < y < 1,$$

$$u(x,0) = 0, \quad u(x,1) = 0, \quad 0 < x < 1,$$

$$u(0,y) = y, \quad u(1,y) = (1-y), \quad 0 \leq y \leq 1.$$

Prove that $u(x,y)$ is C^∞-function for $0 < x < 1$.