TEST: "Introduction to partial differential equations", 04.12.2006

Lecturer: Valery Serov

1) Solve the initial value problem and determine the values x_1 and x_2 for which it exists:

$$x_1 \partial_1 u(x_1, x_2) + u(x_1, x_2) \partial_2 u(x_1, x_2) = x_2, \quad u(x_1, 0) = x_1.$$

2) Prove that the following locally integrable function:

$$G(x, k) = \frac{e^{ik|x|}}{2ik}, \quad k > 0$$

is the fundamental solution of the operator $\frac{d^2}{dx^2} + k^2$ on $\mathbb{R}$.

3) Prove that the Gaussian kernel $K_t(x)$ satisfies

$$K_t(x) \to \delta(x), \quad t \to +0$$

in the sense of distribution in $\mathbb{R}^n$.

4) Solve by Fourier method the heat conductor problem in a bar of lenth 5 cm for which diffusivity is 1. Suppose that the ends of the bar are insulated and the initial temperature distribution of the bar is $f(x) = x^2$. Find the steady state temperature distribution of the bar.

5) Solve by Fourier method the following boundary value problem with Cauchy data for one-dimensional wave equation:

$$u_{tt}(x, t) = u_{xx}(x, t), \quad 0 < x < \pi, \quad t > 0$$

$$u_x(0, t) = 0, \quad u(\pi, t) = 0, \quad t > 0$$

$$u(x, 0) = \sin x, \quad u_t(x, 0) = 0, \quad 0 < x < \pi.$$

6) Solve by Fourier method the following boundary value problem for two-dimensional Laplace equation on the rectangle:

$$\Delta u(x, y) = 0, \quad 0 < x < 2, \quad 0 < y < 1,$$

$$u(x, 0) = 0, \quad u(x, 1) = 0, \quad u(0, y) = 1 - y, \quad u(2, y) = 0.$$

Can we differentiate the series term by term inside of this rectangle? How many times?