Optimization theory Exam, 23.05.2011

1. Two floors and five floors buildings are constructed on a plot with an area of 42000 m^2. Due to the properties of the soil, the need for adequate foundations greatly increases the cost of higher building. A builder has a budget of 90 million euros and 4500 personmonths of workers.

   (a) How many two story and five story buildings should be build, if the number of apartments on this plot is to be maximized? A two story building can have 12 apartments and a five story building 30 apartments. The remaining information is contained in the following table. *(4p)*

	5 floors	2 floors
costs in euros	3 millions	1 million
personmonths	120	60
area in m^2	800	600

   (b) Formulate the problem in (a) to the standard form of linear programming and give its dual problem. *(2p)*

2. (a) Define the following concepts:
   (i) supporting hyperplane.
   (ii) extreme point of a convex set.
   (iii) convex function.

   (b) Suppose that $X \subset \mathbb{R}^n$ is a convex set and let $f \in C^1(X)$. Show that if

   $$f(x) + \nabla f(x)^T (y - x) \leq f(y), \quad x, y \in X,$$

   then f is convex.

3. Consider the geometric program

   $$\frac{1}{t_1 t_2} + t_1 + t_2 = \min!\,, \quad t_1, t_2 > 0.$$

   Give the dual program and solve the original program by using the dual.

4. In constrained convex optimization we study the problem

   (P)
   $$\begin{cases} f(x) = \min! \\ g_i(x) \leq 0,\ 1 \leq i \leq m \\ x \in X. \end{cases}$$

   (a) Explain, what is meant when we say that problem (P) is consistent or superconsistent?

   (b) Show that the domain of function $z \to MP(z)$ is convex.

   (c) Let

   $$\begin{cases} \sqrt{x^2 + y^2} = \min! \\ x + y \leq 0. \end{cases}$$

   Give the problem $P(z)$ and the function $MP(z)$.