Optimization Theory

Exam, 24.01.2011

1. An oil refinery uses heavy and light crude oil as a raw material. Light crude oil costs \$55/barrel and heavy crude oil costs \$50/barrel. The refinery produces gasoline, heating fuel and jet fuel (final products) of the raw material. The amounts of final products made of one barrel of raw material are indicated in the following table: (e.g. one barrel of light crude oil is needed to produce 0,3 barrels of gasoline, etc.)

	gasoline	heating fuel	jet fuel
light crude oil barrel	0,3	0,2	0,3
heavy crude oil barrel	0,3	0,4	0,2

   The refinery has agreed to deliver 900000 barrels of gasoline, 800000 barrels of heating fuel and 500000 barrels of jet fuel.

   (a) How many barrels of light and heavy crude oil the refinery needs, if they want to fulfill their agreement with minimum costs.

   (b) Give the standard form for the problem (a) and formulate the corresponding dual problem.

2. Let A be an $m \times n$ matrix of rank m, $m \leq n$. Consider the standard problem of the linear programming:

$$\begin{aligned}&\text{minimize } c^T x\\ &\text{subject to } Ax = b,\ x \geq 0.\end{aligned}$$

   Prove:

   (a) if there is a feasible solution, there is a basic feasible solution;

3. Calculate the height h and radius r of a cylindrical can of fixed volume V_0, when the cost of the can is minimized. The cost of the top and bottom of the can is c_1 cents/cm^2 and the cost of the side of the can is c_2 cents/cm^2.

4. Consider the SD-method. Show, that

   (a) the sequence of iterations is orthogonal, i.e.

$$\left(x^{(k+1)} - x^{(k)}\right) \cdot \left(x^{(k+2)} - x^{(k+1)}\right) = 0.$$

   (b) if $\nabla f(x^{(k)}) \neq 0$, then $f(x^{(k)}) > f(x^{(k+1)})$.

5. Consider the following program

$$\text{(P)} \qquad \begin{cases} \text{Minimize } f(x) = x^2 - 2x \\ \text{subject to } 0 \leq x \leq 1. \end{cases}$$

(a) Sketch the graphs of the Absolute Value and Courant-Beltrami Penalty Terms for (P).

(b) For each positive integer k, compute the minimizer x_k of the unconstrained objective function $P_k(x)$ with the Courant-Beltrami Penalty Term.

(c) For each positive integer k, compute the minimizer x_k of the unconstrained objective function $F_k(x)$ with the Absolute Value Penalty Term.